A methodological critique of claims for a general factor of personality

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Outline

1. Introduction
   - Factor models of ability and personality analyze the personality sphere
   - It has been claimed that personality inventories show a general factor analogous to that of the ability domain
2. What is a general factor?
   - General, group and specific factors
   - Ability items have a positive manifold and show a general factor, g
   - Evaluating the size of a general factor: Coefficients ω_h and β
   - Expected values of ω_h and β as number of factors varies
3. Claims for a general factor of personality
   - Basic data sets claim to show a gfp
   - These claims for a GFP reflect misunderstanding of what a general factor is
4. Further problems and Conclusion
   - Methodological problems in identifying a general factor

Introduction

- Ability measures are all positively correlated (have a positive manifold) and may be represented with a hierarchical structure of lower order factors and one higher order general factor, g.

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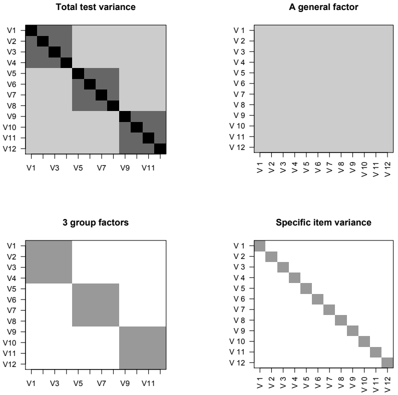
Introduction

- Ability measures are all positively correlated (have a positive manifold) and may be represented with a hierarchical structure of lower order factors and one higher order general factor, g.
- It has been claimed that such a structure is true for non-cognitive personality variables as well.
- These claims, however, reflect a fundamental misunderstanding of how to measure a general factor.
- We review the claims, discuss the problems, and suggest that there is no general factor of personality.

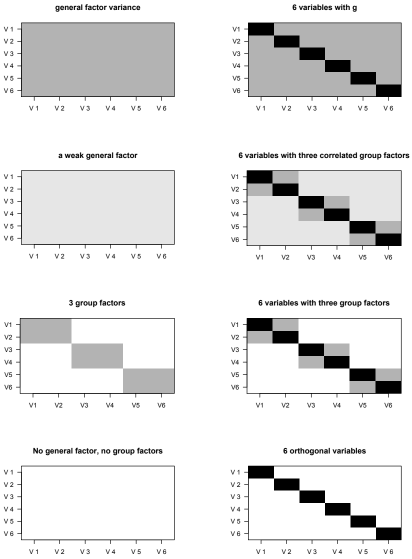
A general factor is analogous to test reliability.



A general factor represents variance common to all variables. Analogous to classical reliability theory of a unifactorial test.

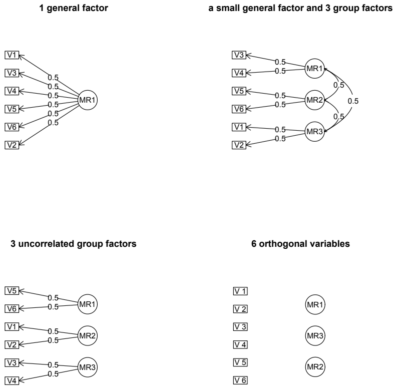
- Total test variance may be decomposed into independent factors
- Test variance = general + group + specific + error
- Test covariance can be partitioned into general + group

General factor, group factors, and tests



- The shared variance of multiple personality tests can be decomposed in the same way.
- Consider four cases of six variables with general and group factors
    1. Just a general factor
    2. 3 groups with a small general factor
    3. 3 orthogonal group factors
    4. 0 group factors and no general factor
- The problem: How to estimate the general factor in these various cases?

Factor analyses show structure



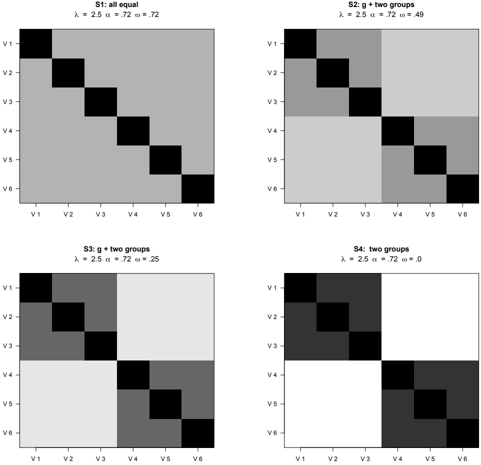
- Consider these four cases of six variables with general and group factors
  1. Just general factor
  2. 3 groups with small general
  3. 3 groups
  4. 0 group factors
- The problem remains: How to estimate the general factor in these various cases?

Several ways have been proposed to estimate a general factor

1. The proportion of variance attributed to the first factor. - NO
   - This is insensitive to structure of tests
2. The correlations between the lower level factors - NO
   - Fails to examine how well the lower level factors account for the tests
3. Factor loadings of a higher order factor on the lower order factors - NO
   - Fails to examine how well the lower level factors account for the tests
4. Amount of test variance accounted for by a 2nd or 3rd order factor - YES
   - ω_h discussed by McDonald (1999) and others (Revelle & Zinbarg, 2009) estimates general factor saturation.

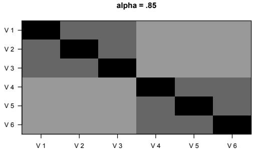
In that hierarchical structure has been discussed in cognitive psychology for 100 years, it is worth considering that solution.

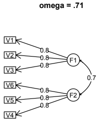
The first factor $\neq$ a general factor

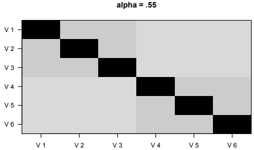


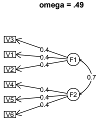
- All sets of 6 items have equal average correlation (.3)
- But differ in the patterning of correlations
  - S1 $r_w = .30 \quad r_b = .3$
  - S2 $r_w = .45 \quad r_b = .2$
  - S3 $r_w = .60 \quad r_b = .1$
  - S4 $r_w = .75 \quad r_b = .0$
- All 1st eigen values = 2.5
- All coefficient $\alpha = .72$
- general factor saturation $\omega_h = .72 \quad .49 \quad .25 \quad .00$

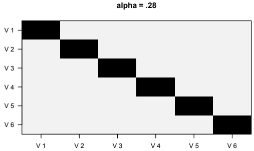
The between factor correlation does not index a general factor

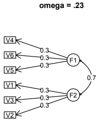






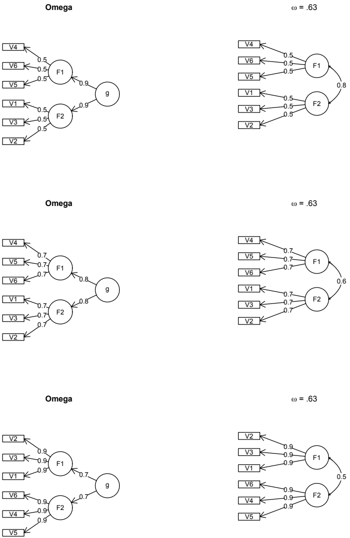






- Between factor correlation reflects ratio of between cluster to within cluster covariances
  - $r_b = .40 \quad r_w = .60$
  - $r_b = .15 \quad r_w = .225$
  - $r_b = .05 \quad r_w = .075$
- Identical interfactor correlations .66 $\neq$ non-identical g saturation $\omega_h = .71 \quad .49 \quad .23$

The higher level factor loading does not index the general factor saturation



- Indeed, it is easy to create cases where varying the between factor correlations or higher level loading does not vary the g saturation
  - $r_{f_1 f_2} = \bar{r_b}/\bar{r_w}$
  - $\omega_h = n^2 \bar{r_b}/V_t$ where
  - $V_t = n^2 * \bar{r_b} + k * m^2(\bar{r_w} - \bar{r_b}) + n(1 - \bar{r_w})$
  - n items, k groups, and m items within each group.
- All $\omega_h = .63$

Hierarchical solutions are typical in the ability domain

- Cognitive ability tests have a positive manifold (are all positively correlated).
- Factor analyses of ability produce correlated lower order factors.
- These factors may, in turn, be factored.
- Example data sets available in R and discussed today include
  - 9 tests from Thurstone and Thurstone (1941)
  - 9 tests from Holzinger (1939)
  - 14 tests from Holzinger and Swineford (1937)
  - 2 sets of 17 tests from Bechtoldt (1961), taken from Thurstone and Thurstone, (1941)
  - 24 tests from Holzinger and Harman (1976)
- Example simulated set from Jensen and Weng (1994)

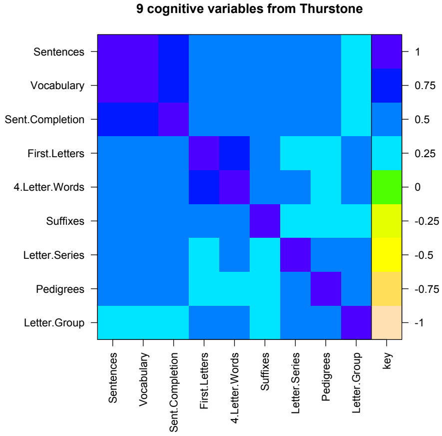
An example from the ability domain

A conventional way to show a correlation matrix

	V1	V2	V3	V4	V5	V6	V7	V8	V9
Sentences	1.00	0.83	0.78	0.44	0.43	0.45	0.45	0.54	0.38
Vocabulary	0.83	1.00	0.78	0.49	0.46	0.49	0.43	0.54	0.36
Sent.Completion	0.78	0.78	1.00	0.46	0.42	0.44	0.40	0.53	0.36
First.Letters	0.44	0.49	0.46	1.00	0.67	0.59	0.38	0.35	0.42
4.Letter.Words	0.43	0.46	0.42	0.67	1.00	0.54	0.40	0.37	0.45
Suffixes	0.45	0.49	0.44	0.59	0.54	1.00	0.29	0.32	0.32
Letter.Series	0.45	0.43	0.40	0.38	0.40	0.29	1.00	0.56	0.60
Pedigrees	0.54	0.54	0.53	0.35	0.37	0.32	0.56	1.00	0.45
Letter.Group	0.38	0.36	0.36	0.42	0.45	0.32	0.60	0.45	1.00

9 cognitive variables from Thurstone and Thurstone (1941). Used by McDonald (1999) and others for demonstrations of coefficients ω_h and ω_t

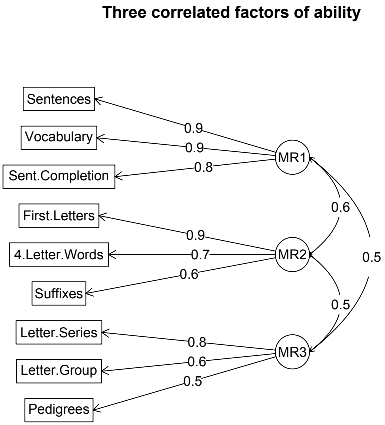
An alternative way to show correlation matrices



9 cognitive ability tests from Thurstone and Thurstone (1941) is a classic data set for showing hierarchical structure. Graphic presentations of correlations are clearer.

- Color indicates size of correlation.
- All correlations are $> .29$ with a mean of .54
- Three lower level (group) factors may be seen

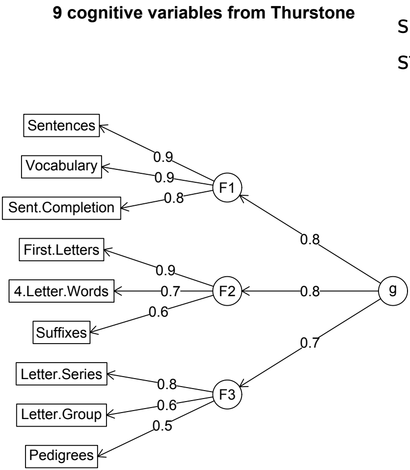
Correlated factor structures: Evidence for g



These 9 ability tests may used to show concept of correlated structure

- Clear lower level three factor solution
- Correlations between factors represent a g factor
- Apply a second order factor analysis to these correlations

Hierarchical factor structure: Yet another way of looking at g

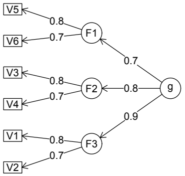


These 9 ability tests may used to show concept of hierarchical structure

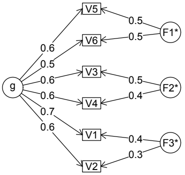
- A hierarchical structure represents at least two levels of factors
- Clear lower level three factor solution
- Correlations between factors accounted for by a g factor

McDonald's *omega*$_h$ and the bifactor model

A hierarchical solution



A bifactor solution

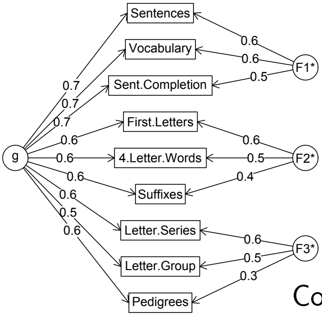


- A hierarchical solution
  - Extract 3 factors and transform obliquely
  - Factor the factor correlations
- A Bifactor solution (Holzinger and Swineford, 1937) may be found using the Schmid-Leiman transformation
  - Find the general loading from hierarchical loading x group loading
  - Remove from group to find residual factor loading
- EFA analysis done in omega function with *psych* package in R.
- CFA analysis done with *sem* package in R.

Several different statistics for evaluating the size of a general factor

$$\omega = .74$$

9 cognitive variables from Thurstone



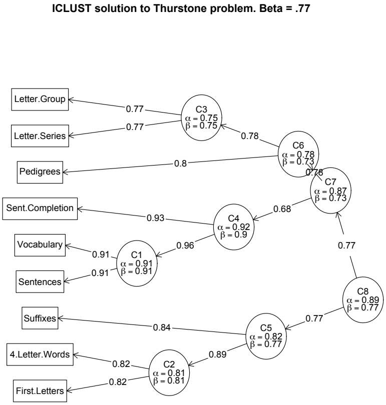
Coefficient ω_h is the ratio of general factor variance to total variance.

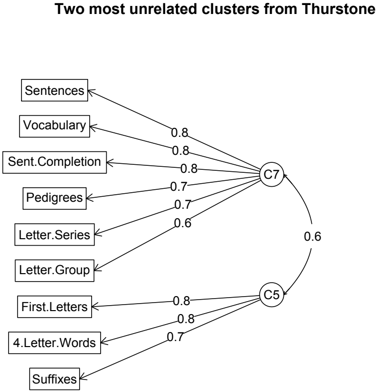
- General factor variance is squared sum of general factor loadings.
- Total variance is just the sum of the variance/covariance matrix.
- Can also consider the amount of each item/test variance accounted for by the general factor loading
- Examine the mean and variance of this percentage

Coefficient β is the worst split half reliability and is a cluster based estimate of general factor saturation

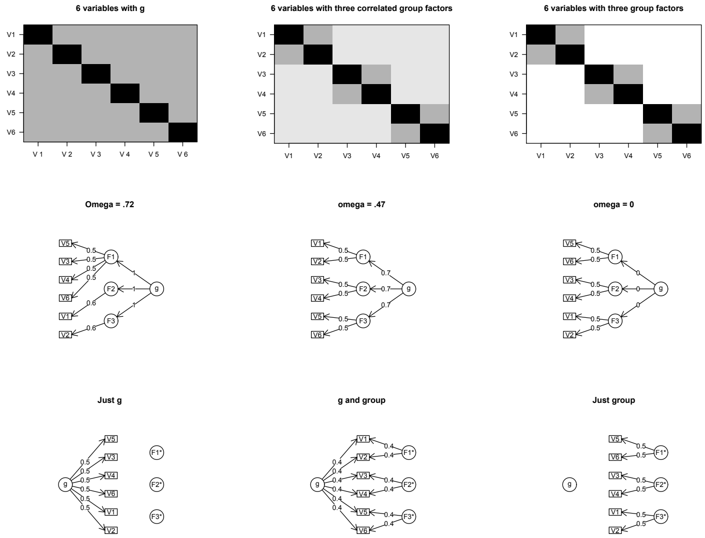
β also estimates general factor saturation

β is the worst split half reliability and is found using hierarchical cluster analysis (ICLUST). For Thurstone data set, the two most unrelated parts correlate .61 and thus *beta* is .77.



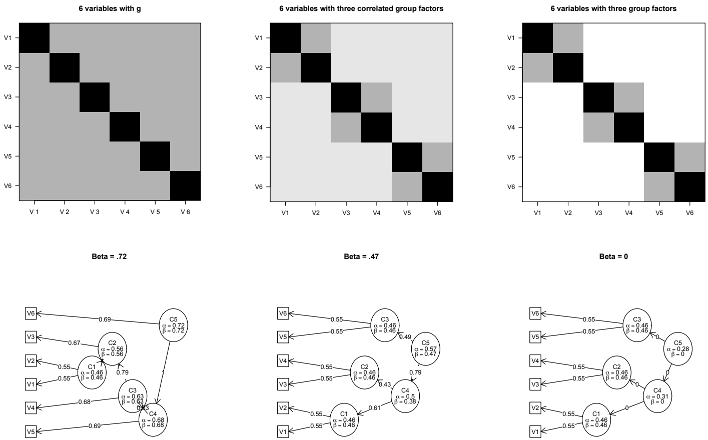


Apply omega to our examples



The hierarchical structures appear similar, we need to examine the coefficients. Also can examine the loadings in the Schmid-Leiman solution.

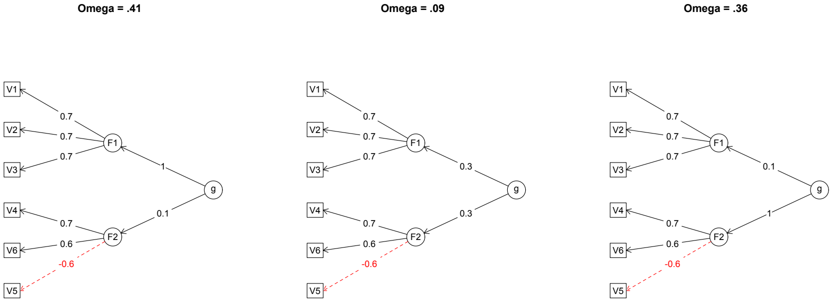
Apply ICLUST to our examples



The hierarchical structures appear similar, we need to examine the beta values

Unfortunately, ω_h for a 2 factor solution is underidentified in EFA

For two lower level factors, the higher level model is under-identified and depends upon external assumptions.



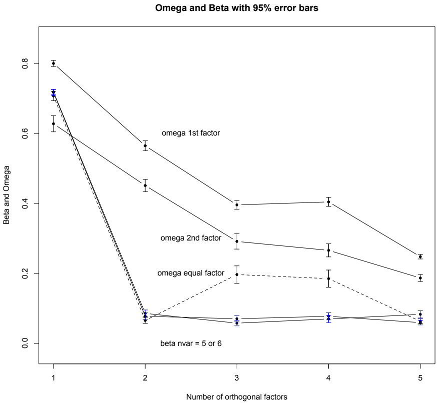
Although the structural models appear identical and goodness of fit statistics are identical, the general factor solution differs drastically. $.09 < \omega_h < .41$. β, based upon a simpler model, is .09.

Estimating a general factor for 5 tests is under-identified in EFA

- For 5 variables (e.g., the "Big 5"), only 2 lower order factors can be extracted using structural modeling software.
  - The degrees of freedom for 2 factors and 5 variables is 1
  - The degrees of freedom for 3 factors and 5 variables is -2
- This means that to apply CFA or SEM to "Big 5" problems is limited to 2 factor solutions
- But 2 lower level factors are not identified for a higher order solution without some additional constraints.
- The effect of these constraints may be seen by simulation

Omega and Beta vary as the number of underlying factors

Simulations of 1 .. 5 factors causing the correlations of 5 variables.

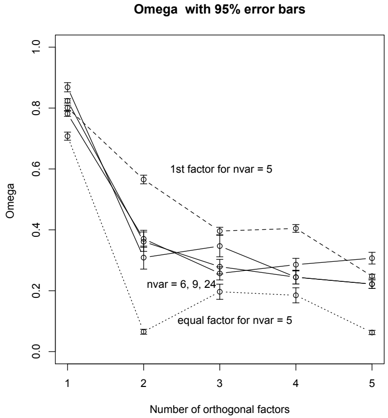


- number of (orthogonal) factors = 1 ... 5
- number of tests = 5
- replications = 100
- ω_h estimated with 2 factors
  - using 1st as g
  - using 2nd as g
  - using $\sqrt{\phi}$ as g
- β found using ICLUST

Results show that ω_h defined by 1st or 2nd factor is highly biased.

Omega varies as the number of underlying factors

Repeat this simulation for 5, 6, 9 and 24 variables

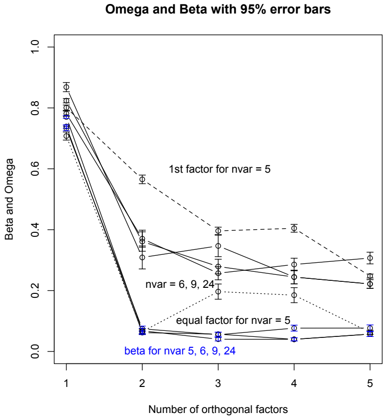


- number of factors = 1 ... 5
- number of tests = 5, 6, 9, 24
- replications = 100
- ω_h estimated with 2 factors
  - using 1st as g
  - using 2nd as g
  - using $\sqrt{\phi}$ as g
- otherwise ω_h estimated with 3 factors

ω_h is positively biased for all cases of factors > 1

Omega and Beta vary as the number of underlying factors

Include ICLUST estimates of β for 5, 6, 9 and 24 variables



- number of factors = 1 ... 5
- number of tests = 5, 6, 9, 24
- replications = 100
- ω_h estimated with 2 factors
  - using 1st as g
  - using 2nd as g
  - using $\sqrt{\phi}$ as g
- otherwise ω_h estimated with 3 factors
- β found using ICLUST
- β does not show the same bias.

Omega is positively biased if items can be reversed

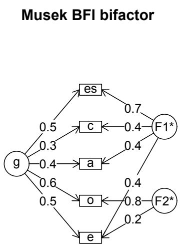
- Although ω_h is an excellent estimate of general factor saturation when there is one factor, ω_h is positively biased in the case of multiple factors if items can be reversed.
- Particularly with < 6 variables when we must use just 2 lower level factors.
  - Need to search for solution that minimizes ω_h
  - Not the solution that maximizes ω_h
- β might be preferred in case of few variables or when the real structure has > 1 factor.
  - That is, ω_h should be used for evaluating how well a unifactorial test measures one thing.
  - But β should be used to determine how general factor saturation in presence of multiple factors.
  - Other tests for the number of factors should be used as well.

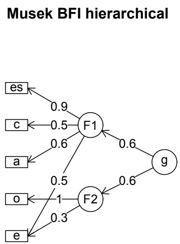
Claims for a general factor

- Musek (2007)
  - using BFI (John, Donahue & Kentle, 1991), IPIP (Goldberg, 1999) and Big 5 Observer (Caprara, Barbaranelli & Borgogni, 1994).
- Rushton & Irwing (2008)
  - Digman (1997)
  - Meta analysis of Big 5 inventories (Mount, Barrick, Scullen & Rounds, 2005)
- Rushton, Bons & Hur (2008)
- Rushton & Irwing (2009a,c)
  - Comrey Personality Scales (Comrey, 1995, 2008)
  - MMPI2 (Helmes, 2008)
  - Multicultural Personality Questionnaire (Van der Zee & Van Oudenhoven, 2001)
  - Multidimensional Personality Questionnaire (Tellegen, 1982; Tellegen & Waller, 2008)
- Erdle, Irwing, Rushton & Park (2010) Web based BFI
- Zawadzki & Strelau (2010)

Different methods used in these various studies for estimating the general factor

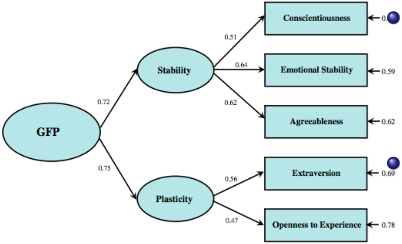
- Musek (2007) used size of first principal component
- Rushton & Irwing (2008) reports correlations between lower level factors
- Rushton & Irwing (2009a,c) use Structural Equation Modeling with questionable models and emphasize factor loadings of lower level factors on g without examining the g effect on basic variables.
- Rushton & Irwing (2009b) uses SEM and and emphasizes the fit to "reliable variance" meaning the lower level factors, but not the basic tests themselves.
- Erdle et al. (2010) uses SEM of "reliable variance"
- Zawadzki & Strelau (2010) uses SEM (and is potentially the best of the studies, because it has structures for peer ratings as well as self ratings)





- Musek (2007)
  - Big Five Inventory
- Analysis based upon 1st eigen value
- Actually agrees with ω_h estimate
- But this will not normally be the case

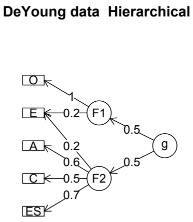
Rushton & Irwing (2009b) and the DeYoung data

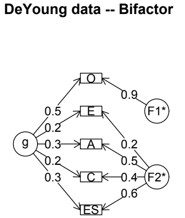


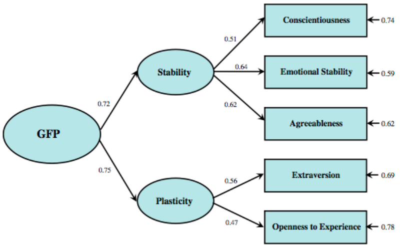
- Refers to reliable variance as if the tests themselves are not reliable
- DeYoung reliabilities are actually $> .8$

- Rushton & Irwing (2009b)
  - Data from 16 studies by Colin DeYoung (N $= 6412$)
  - Big Five Inventory
- "The model explains 54% of the variance in DeYoung's factors of Stability and Plasticity, i.e. 54% of the reliable variance. However, because there is substantial error in most of the indicators, this only translates into 16% of the scale level variance.
- SEM based ω is .42
- EFA produces an ω of .25

EFA vs. CFA of DeYoung data

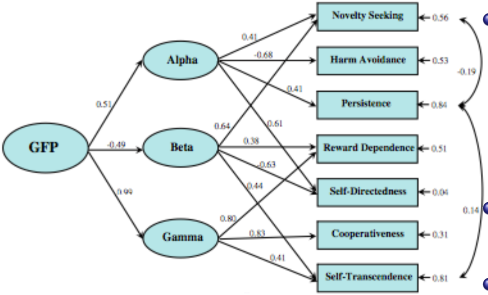






- Rushton reports .54 = product of g factor loadings
- But ω_h from this diagram is .42
- EFA *omega* = .25

Cloninger Temperament and Character Inventory



- "The GFP accounted for 49% of the variance in the three first-order factors and 24% of the total reliable variance."
- ω_h from EFA is .40 and from SEM .38
- Great variability in "general factor" loadings
  - general factor loading are -0.09 -0.49 0.29 0.29 0.98 0.58 0.22

- "Gamma" factor has loading of .99 on general suggesting that the "general" factor is "gamma".

Guilford Zimmerman Temperament Survey

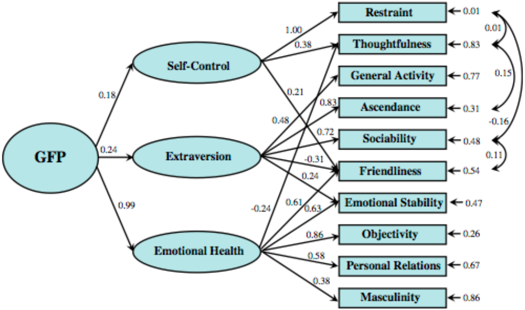


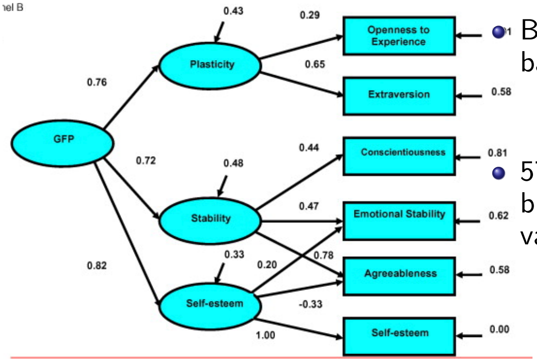
Fig. 2. GZTS third-order confirmatory common factor structure.

- But Emotional Health loads .99 on the general factor and the other two higher level factors have loadings of .18 and .24.

- Rushton & Irwing (2009b) report that "The GFP accounted for 36% of the variance in the three first-order factors and 21% of the total reliable variance.

Erdle et al. (2010)

"The model explains 57% of the variance in the factors of Stability and Plasticity, i.e. 57% of the reliable variance. However, because there is substantial error in most of the indicators, this only translates into 15% of the scale level variance."



- Based upon 628,640 web based participants
  - Given the BFI and a 1 item self esteem question.
- 57% of the reliable variance but 15% of the scale level variance
  - And that is the problem.
  - Factors are not "the reliable variance"

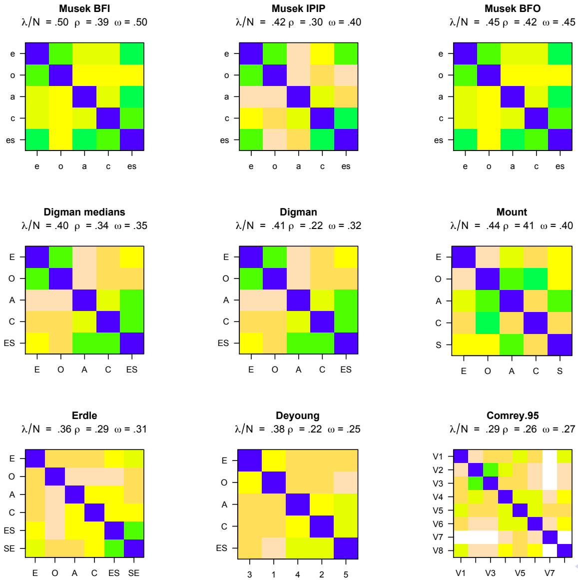
The g-factor is similar but not identical across studies

Table: Factor congruence coefficients for g may be found for those studies with the same variables. Generally, factor congruences > .95 are expected for a good match. (For instance, the two Bechtoldt data sets of ability have g-factor congruences of .99 and the two Comrey data sets have a factor congruence for g of 1.0.)

Study	BFI	IPIP	Digman	Erdle	DeYoung
Musek BFI	1.00				
Musek IPIP	.88	1.00			
Digman	.95	.96	1.00		
Erdle	.88	.94	.98	1.00	
DeYoung	.98	.82	.89	.79	1.00

	Method 1		Method 2	Method 3			
	C_1/N	Λ_1/N	r_Λ	Λ_g/N	ω_h	R^2	R/M
Rushton's Analyses							
Digman's Medians	.40	.27	.33	.14	.35	.35	.45
Mount's Meta Analysis	.44	.31	.41	.17	.40	.37	.44
Comrey Scales	.29	.19	.26	.08	.27	.31	.41
MMPI	.39	.35	.15	.16	.37	.41	.49
MPQ	.46	.33	.25	.15	.31	.35	.33
Musek's Analyses							
Big Five Inventory	.50	.39	.38	.23	.50	.52	.50
IPIP	.42	.29	.30	.17	.40	.46	.40
Big Five Observer	.45	.32	.42	.20	.45	.45	.45
Cognitive Ability Data Sets							
Thurstone's 9	.54	.48	.55	.40	.74	.74	
Thurstone and Bechtoldt's 17	.37	.34	.29	.28	.72	.78	
Holzinger's 14	.37	.32	.29	.28	.71	.69	
Brigham and Thurstone's 9	.56	.51	.56	.49	.85	.89	
Holzinger and Harman's 24	.34	.31	.38	.22	.65	.66	
Means							
Rushton & Musek	.42	.31	.31	.16	.38	.40	.43
Cognitive Ability	.44	.39	.41	.33	.73	.75	

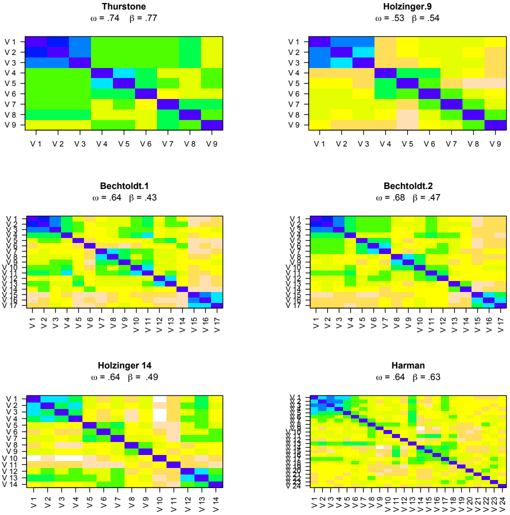
Graphic representation of these data sets



- Musek
  - BFI $\omega = .50$
  - IPIP $\omega = .40$
  - BFO $\omega = .45$
- Rushton
  - Digman medians $\omega = .35$
  - Digman $\omega = .32$
  - Mount $\omega = .40$
  - Erdle $\omega = .31$
  - DeYoung $\omega = .25$
  - Comrey.95 $\omega = .27$

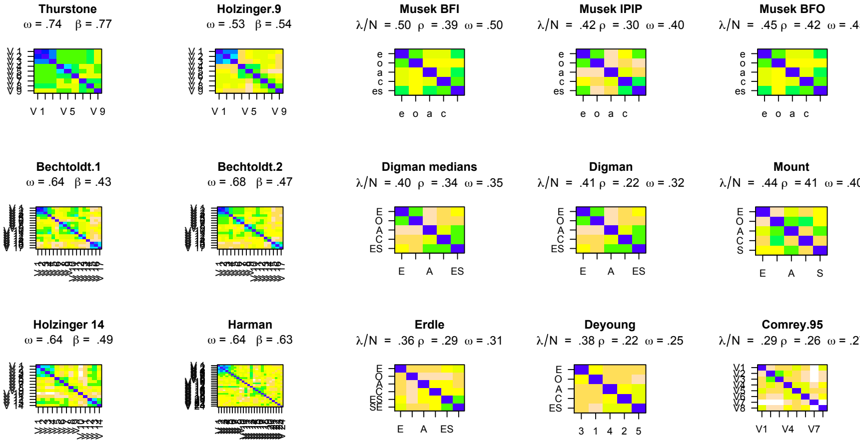
Compare with correlations of ability

6 cognitive data sets available in R



- Six data sets from R
  - Thurstone and Thurstone (1941)
  - Holzinger (1939)
  - Bechtoldt (1 and 2) (1961)
  - Holzinger and Swineford (1937)
  - Harman (1976) 24 mental tests
- The results
  - Generally larger background (general) level.
  - Some obvious clumping

Compare Cognitive to Personality



- Compare 6 cognitive data sets to 9 personality data sets
  - Cognitive are columns 1-2
  - Personality are 3-5

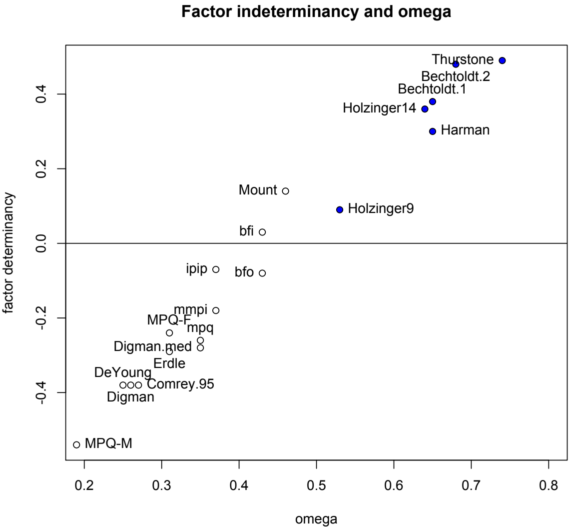
Methodologial problems with a identifying a general factor

- The problem of factor indeterminacy
  - Factor score estimates are compatible with many different factor solutions
  - Indeterminacy varies inversely as factor saturation
  - Most personality data sets have an indeterminate general factor
- The problem of item reversals
  - General factor implies positive manifold and unipolar items
  - Personality items tend to be bipolar

The problem of factor indeterminacy

- Factor loadings are beta weights that predict the tests from the factors.
- To estimate factor scores we need to find weights to predict factors from tests.
- $\mathbf{W} = \mathbf{F}\mathbf{C}^{-1}$ and the multiple R^2 between the factors and their estimate is $\mathbf{R}^2 = diag(\mathbf{W}\mathbf{F})$.
- Different latent factors can provide equally good fits to factor score estimates.
- The minimum correlation between two factors that would produce the factor score estimates is $R_{min} = 2 - R^2$.
- If this minimum < 0 then the factor is completely indeterminate for two different factors could be negatively correlated but still produce the same factor score estimate.
- This is particularly a problem if the factor loadings are not very big.

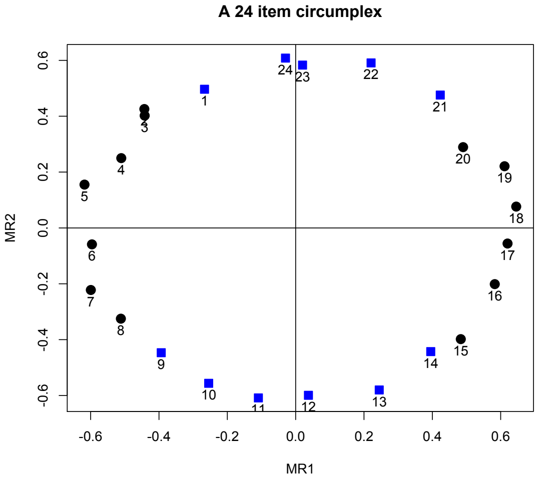
The general factor of personality is indeterminate.



- All cognitive batteries have $\omega > .5$ and $R_{min} > .0$
- All personality battery have $\omega < .5$
- 11 have $R_{min} < .5$ and are completely indeterminate

$1 \neq 2 \neq 3$: The problem of item reversals

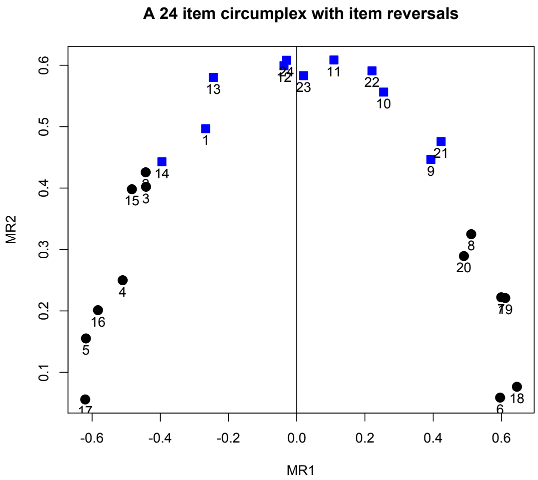
A two dimensional circumplex clearly has no general factor.



- Two orthogonal bipolar dimensions show no general factor.
- $omega_h = 0.0$
- But, what happens if we reverse key some items?

$1 \neq 2 \neq 3$

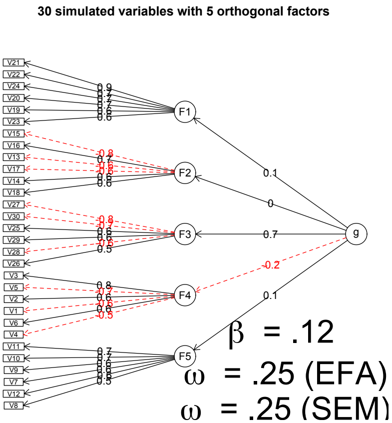
A "general" factor created by item reversals



- items 6 … 17 reversed keyed.
- There appears to be a general factor!
- $\omega_h = .72$
- This is a problem for personality variables in which direction is arbitrary. Allowing sign reversals will falsely indicate a general factor.

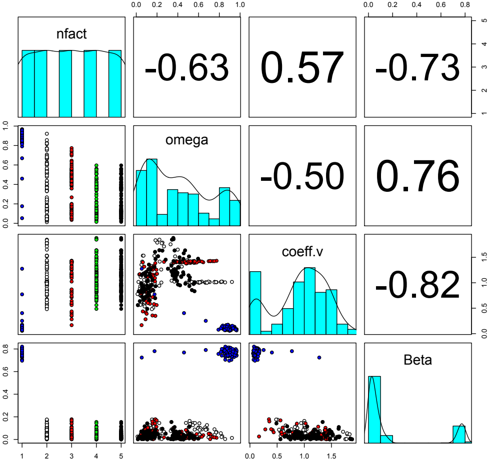
$1 \neq 5$

Simulation of 5 orthogonal factors (+ noise)



- Simulate 30 variables with 5 major bipolar and 15 trivial noise factors.
    - ω_h found by exploratory factor analysis for 5 factor solution is .25
    - General Factor saturation from SEM is .25
    - β from cluster analysis is .12
- What is the right answer? 0
- Why does this happen? Items may be reversed.

Estimates of omega are positively biased if items may be reversed



- 100 simulations of 1, 2, 3, 4 or 5 factor data sets
  - omega found by EFA
  - beta found by ICLUST
  - cv is coefficient of variation of factor loadings
- if number of factors is > 1 then real value is 0 and bias is roughly $1/\text{number of factors}$
- β has much less bias

Do confirmatory models work better?

- Our analyses were done using Exploratory Factor Analysis (EFA).
- Do confirmatory models work better?
  - Confirmatory Factor Analysis (CFA)
  - Structural Equation Modeling (SEM)
- No. Not if the same mistakes in logic are applied.
  - Goodness of fit $\neq$ confirmation of gfp adequacy
  - Can not just find g loadings on higher level factors
  - Nor are the g loadings on lower order factor appropriate
  - Need to estimate ω_h
- Sometimes. If care is not applied in the EFA.
  - Too few lower level factors for identification
  - One lower order factor equated with g inflates estimate of ω_h

What would a general factor look like?

- All correlations should be positive
- Loadings of all items on the g factor should be roughly the same
- General factor score estimates should have $R^2 > .5$ with g factor
- g factor saturations should be greater than expected by chance!

Conclusions

- Are scales measuring the Big 5 orthogonal? No
- Are there higher order factors of the Big 5 scales? Yes.
  - Although the "Big 5" might represent orthogonal factors, the representative scales certainly do not.
  - A substantive interpretation has not been offered by us, but has by others.
- Is there a general factor of personality?
  - Not one that is worth talking about.
  - If there is a general general factor, it is indeterminate.

Conclusion

- Claims for a general factor of personality are not supported by appropriate analysis.
- The reported "general factors" of personality are not general.
- "general factors" with ω_h of .2-.5 may be obtained even when the basic data are formed from nearly orthogonal factors.
- GFP results more likely reflect 2-3 broad factors of personality.
- Appropriate psychometrics may be done with free, but powerful software. e.g., all analyses and graphics were done in the open source statistical program R using the *psych* and *sem* packages.
- For more information, visit the Personality Project website `http://personality-project.org`

Further information

5. Finding ω_h using EFA
6. Determining the number of factors
   - Ability tests
   - Personality inventories
7. Various statistics associated with estimating ω_h
8. References

Finding a general factor using EFA

```
Alpha:                 0.89
G.6:                   0.91
Omega Hierarchical:    0.74
Omega H asymptotic:    0.79
Omega Total            0.93
Schmid Leiman Factor loadings greater than  0.2
                  g   F1*   F2*   F3*   h2   u2   p2
Sentences       0.71  0.57              0.82 0.18 0.61
Vocabulary      0.73  0.55              0.84 0.16 0.63
Sent.Completion 0.68  0.52              0.73 0.27 0.63
First.Letters   0.65        0.56        0.73 0.27 0.57
4.Letter.Words  0.62        0.49        0.63 0.37 0.61
Suffixes        0.56        0.41        0.50 0.50 0.63
Letter.Series   0.59              0.61  0.72 0.28 0.48
Pedigrees       0.58  0.23        0.34  0.50 0.50 0.66
Letter.Group    0.54              0.46  0.53 0.47 0.56

With eigenvalues of:
   g  F1*  F2*  F3*
3.58 0.96 0.74 0.71
general/max  3.71   max/min =   1.35
mean percent general =  0.6    with sd =  0.05 and cv of  0.09
```

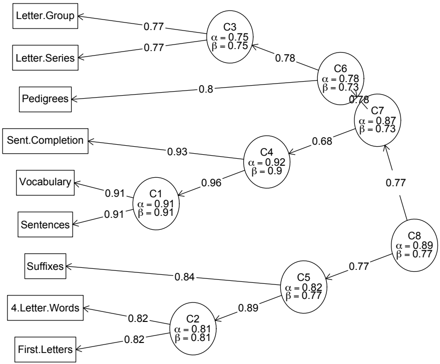
Additional output

```
The degrees of freedom are 12  and the fit is  0.01
The number of observations was  213  with Chi Square =  2.82  with prob <  1
The root mean square of the residuals is  0
The df corrected root mean square of the residuals is  0.01
RMSEA and the  0.1 confidence intervals are  0 0 0.023
BIC =  -61.51

Compare this with the adequacy of just a general factor and no group factors
The degrees of freedom for just the general factor are 27  and the fit is  1.48
The number of observations was  213  with Chi Square =  307.1  with prob <  2.8
The root mean square of the residuals is  0.1
The df corrected root mean square of the residuals is  0.16

RMSEA and the  0.1 confidence intervals are  0.224 0.223 0.226
BIC =  162.35

Measures of factor score adequacy
                                                 g  F1*  F2*  F3*
Correlation of scores with factors            0.86 0.73 0.72 0.75
Multiple R square of scores with factors      0.74 0.54 0.52 0.56
Minimum correlation of factor score estimates 0.49 0.08 0.03 0.11
```

β estimates general factor saturation

β is the worst split half reliability and is found using hierarchical cluster analysis (ICLUST). For Thurstone data set, $\beta = .77$

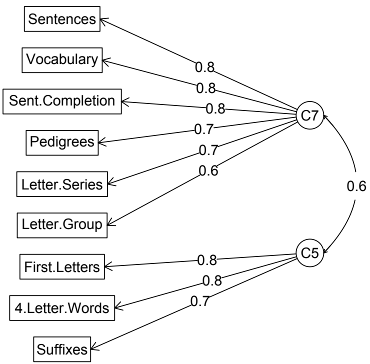
ICLUST solution to Thurstone problem. Beta = .77



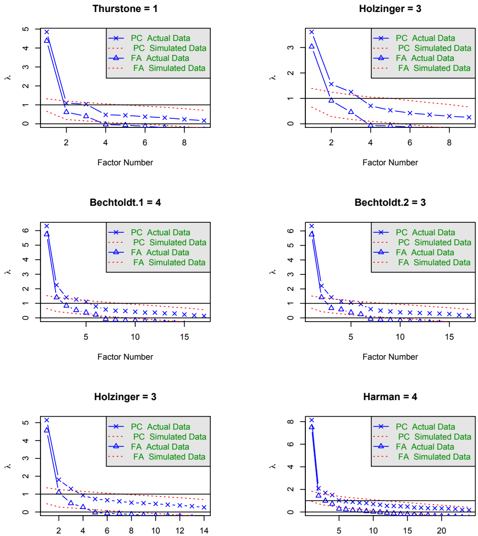
β estimates general factor saturation

β is the worst split half reliability and is found using hierarchical cluster analysis (ICLUST). For Thurstone data set, the two most unrelated parts correlate .61 and thus *beta* is .77.

Two most unrelated clusters from Thurstone

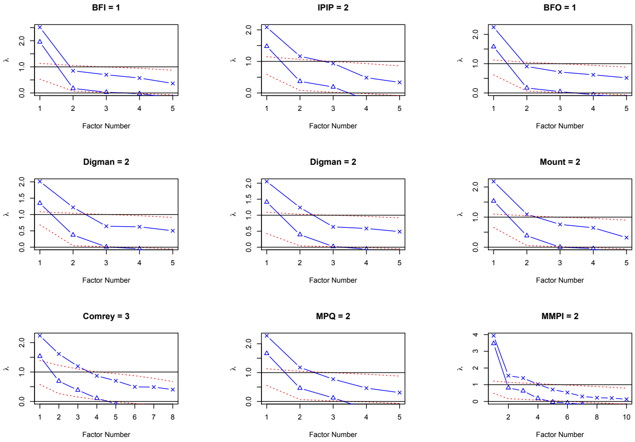


Determining the number of factors through parallel analysis: Case 1: Ability tests



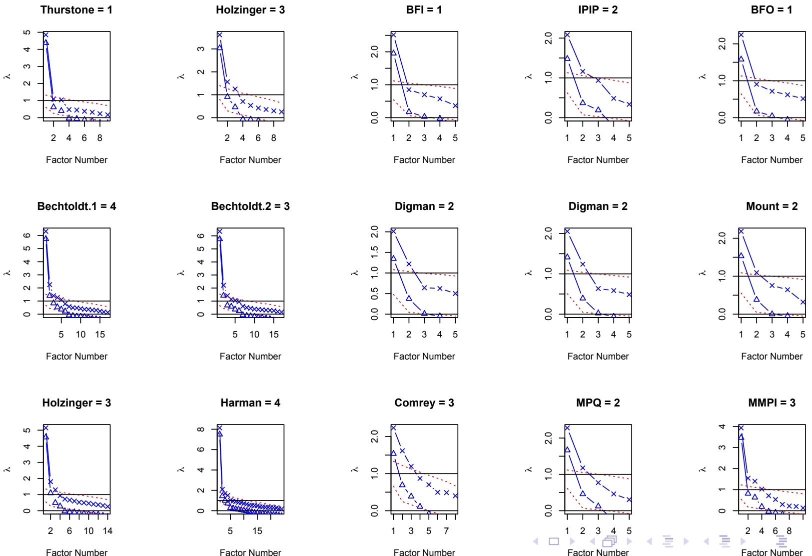
- Parallel analysis compares real matrices with random data matrices of the same rank
  - Plot the scree for components and for factors
  - Compare real scree with random scree
- Although multiple factors are indicated, note the relative size of first factor

Determining the number of factors through parallel analysis: Case 2: Personality inventories

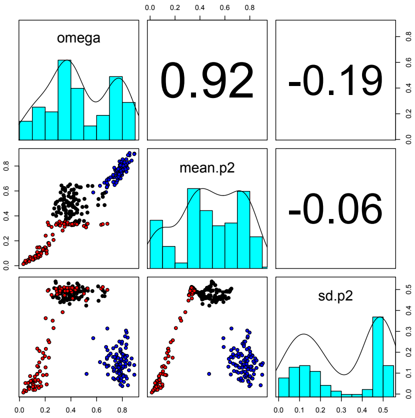


- Musek
  - BFI
  - IPIP
  - BFO
- Rushton and Irwing
  - Digman
  - Mount meta analysis
  - Comrey manual
  - MPQ ?
  - MMPI ?

Determining the number of factors through parallel analysis: Case 1 & 2

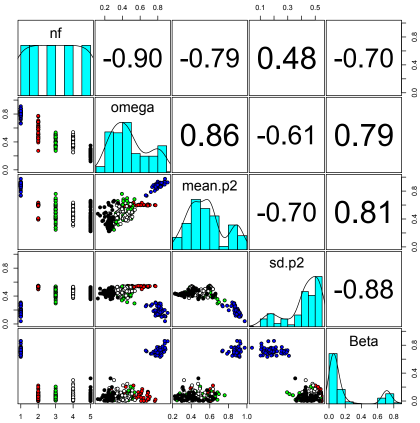


Omega for 1 or 3 factors and 6 variables



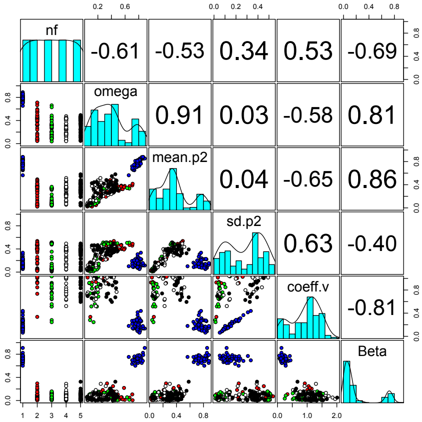
- Simulation of 6 variables with
  - 1 (general) or 3 (orthogonal) factors
  - ω estimated with 2 or 3 factors
- Scatter Plot Matrix (SPLOM) shows
  - distribution of ω_h
  - mean percent g saturation
  - standard deviation of g saturation
- Expected value for ω for 3 underlying (orthogonal) factors when just 2 factors are extracted is .5!

ω_h and β for 1 .. 5 factors and 5 variables



- Simulation of 5 variables with
  - 1 (general) or 2 ... 5 (orthogonal) factors
  - ω_h estimated with 2 factors
- Scatter Plot Matrix (SPLOM) shows
  - distribution of ω_h
  - mean percent g saturation
  - standard deviation of g saturation
  - β = worst split half reliability
- Expected value for ω for 3 underlying (orthogonal) factors when just 2 factors

ω_h and β for 1 .. 5 factors and 6 variables



- Simulation of 6 variables with
  - 1 (general) or 2 ... 5 (orthogonal) factors
  - ω_h estimated with 3 factors
- Scatter Plot Matrix (SPLOM) shows
  - distribution of ω_h
  - mean percent g saturation
  - standard deviation of g saturation
- Expected value for ω for 2 underlying (orthogonal) factors when just 3 factors are extracted is .36!

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