Traditional and modern psychometrics using R: Presented as part of a symposium on Using R in personality research A symposium at the First World Conference on Personality

William Revelle

Department of Psychology
Northwestern University
Evanston, Illinois USA

March 22, 2013

Outline of Part II: Traditional and modern psychometrics using R

- William Revelle : Traditional and modern psychometrics

3. Exploratory Factor Analysis
   - Structure of Mood
   - Factor Extension and Set Correlation as ways of relating multiple domains
4. Classical and IRT approaches to test construction
   - Classical test theory – going beyond α
   - IRT measures of reliability
5. Conclusion

Traditional and modern psychometrics using R

William Revelle Northwestern University
To understand how personality constructs relate to other psychological constructs as well as real world criteria, it is first necessary to develop and assess the reliability and validity of personality scales. The psych package in R has been developed for this purpose. Basic item analysis, factor and cluster analysis, reliability analysis, and item response measures can be done in the psych (Revelle, 2012) package. Two other packages, sem (Fox, 2012) and lavaan (Rosseel, 2012) have been developed to allow confirmatory factor analysis and structural equation modeling. The use of all three of these packages in measuring and evaluating the structure of personality constructs will be demonstrated.

What is the structure of mood?

1. Multiple representations of mood dimensions
   - Positive and Negative Affect (Watson & Tellegen, 1985)
   - Valence and Arousal (Barrett & Russell, 1998)
   - Energetic and Tense Arousal (Thayer, 1978, 2000)
2. Various psychometric solutions
   - Two dimensional simple structure models
   - Two dimensional circumplex models
3. Various problems
   - Unipolar vs. bipolar items (Russell & Carroll, 1999)
   - Item skew (Rafaeli & Revelle, 2006)

Analysis of the Motivational State Questionnaire

The Motivational State Questionnaire (MSQ) was developed to study mood and arousal (Revelle & Anderson, 1997). It included adjectives from a variety of sources. Included in *psych* as the `msq` data set are 75 mood and arousal items given to 3896 subjects over 10 years.

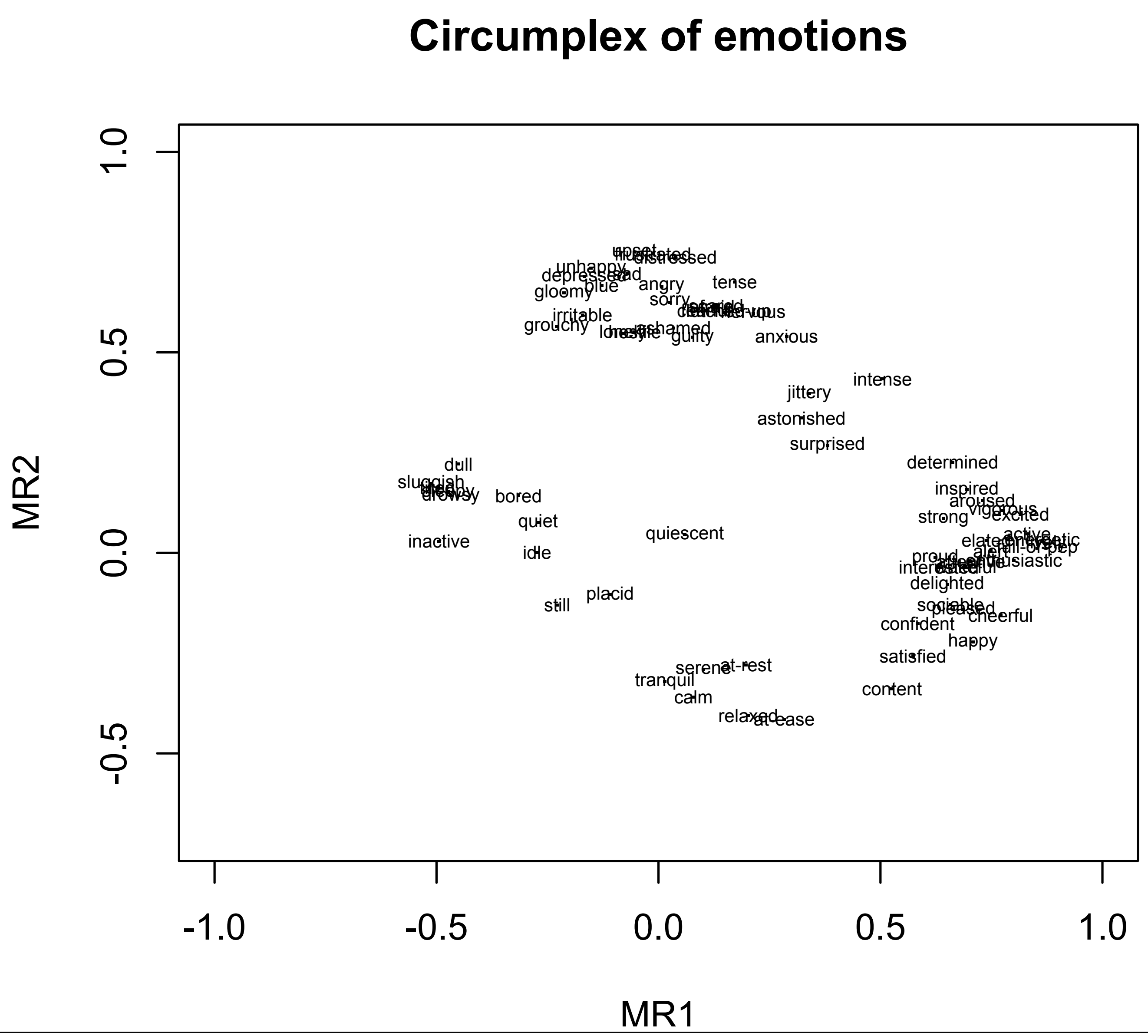
```
> f2 <- fa(msq[1:70],2)
```

```
> plot(f2$loadings,
    xlim=c(-1,1),ylim=c(-.7,1),
    main="Circumplex of emotions",pch='.')
```

```
> text(f2$loadings,
      rownames(f2$loadings),cex=.5)
```

1. Factor analyze the first 70 msq items. Extract two factors.
2. Plot the resulting solution, setting the size of the x and y axes. Use a small plot character.
3. Add labels for each data point. Use a small character size.

Structure of MSQ emotions using Pearson R



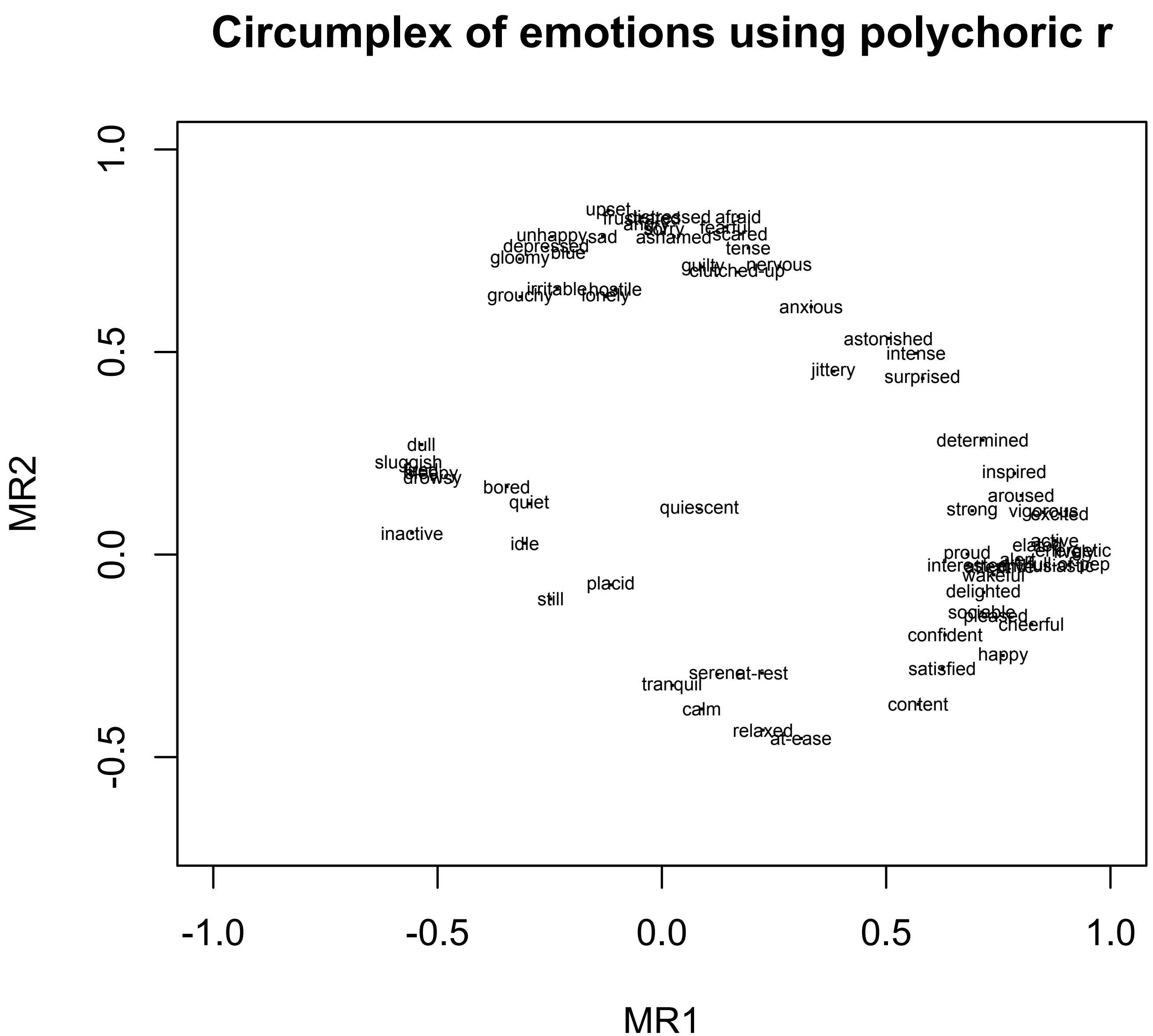
Structure emotion using polychoric correlations

Because the MSQ items were 1-4 we should not treat them as continuous, but rather as categorical. We can find polychoric correlations to compensate for skew.

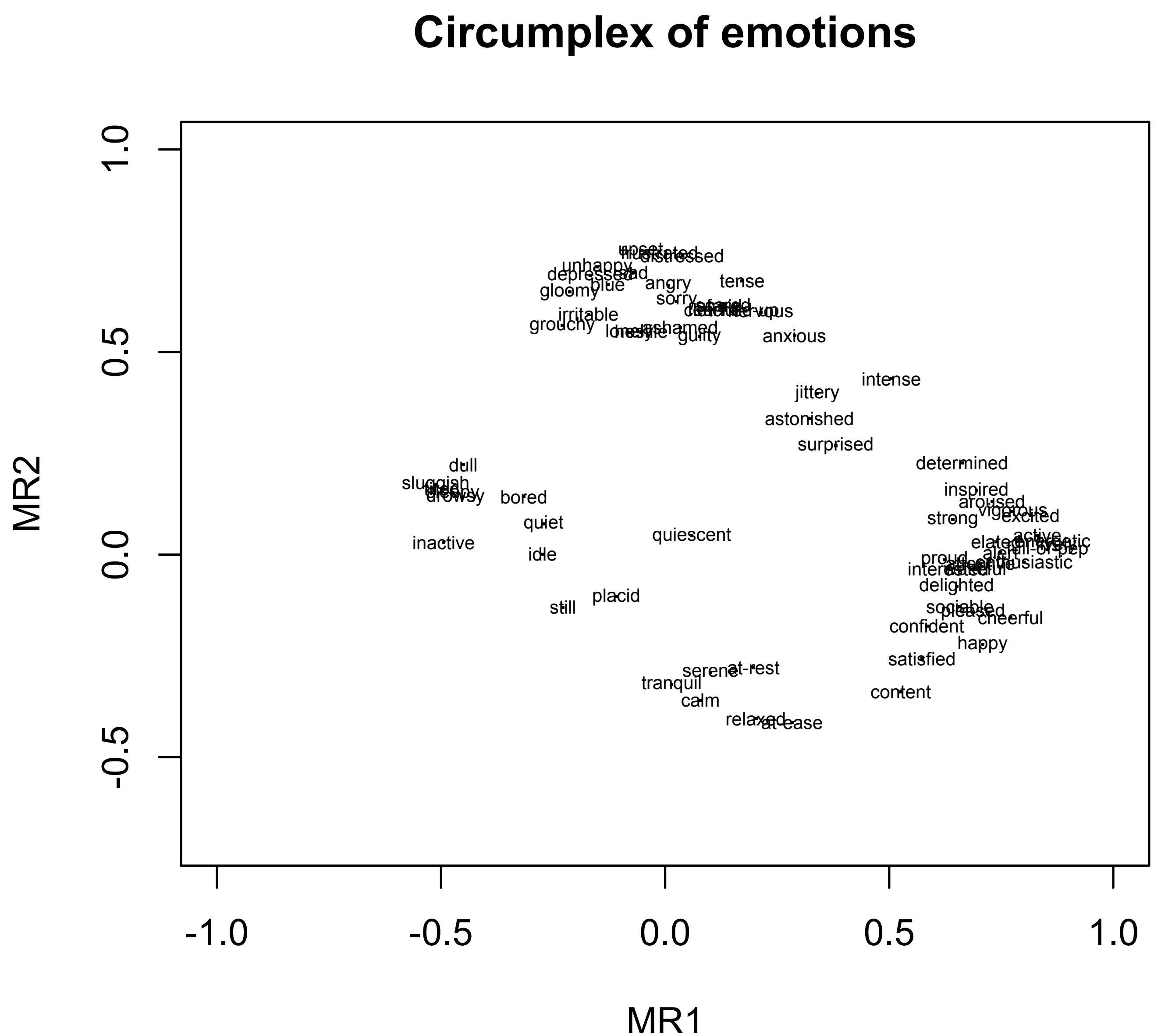
```
> msqR <- polychoric(msq[1:70])

> f2p <- fa(msqR$rho,2)

> plot(f2p$loadings,
        xlim=c(-1,1),ylim=c(-.7,1),
       main="Circumplex of emotions
            using polychoric r",
       pch='.')

> text(f2p$loadings,rownames(f2$loadings),
     cex=.5)
```

1. Find the polychroric correlations of the first 70 msq items
2. Factor analyze the resulting correlations. Extract two factors.
3. Plot the resulting solution, setting the size of the x and y axes. Use a small plot character.
4. Add labels for each data point. Use a small character .

Structure of MSQ emotions using polychoric R



Compare with the structure of MSQ emotions using Pearson R



Compare the 2 solutions in terms of factor congruence

Factor congruence is just the cosine of the angle between the factors:

$$r_c = \frac{\sum_1^n F_{xi} F_{yi}}{\sqrt{\sum_1^n F_{xi}^2 \sum_1^n F_{yi}^2}}.$$

or

$$diag(\mathbf{F_x F_x}')^{-1/2}$$

It may be found using the `factor.congruence` function. We should not just correlate the loadings.

```
> factor.congruence(f2,f2p)

     MR1   MR2
MR1  1.00 -0.04
MR2 -0.06  1.00
```

```
> round(cor(f2$loadings,
              f2p$loadings),2)

     MR1   MR2
MR1  1.00 -0.40
MR2 -0.39  0.99
```

The factors are essentially identical.

Factor Extension and Set Correlation

1. Originally developed by Dwyer (1937) for the case of having completed a factor analysis and then a new variable is introduced.
   - At the time, factoring was hard and time consuming
2. May now be used to extend the factors from one domain into another domain (Horn, 1973).
   - Differs from SEM in that the factors are estimated in the first domain and are not changed with the addition of the second domain
3. Another technique for relating two domains is "Set Correlation" as discussed by Cohen, Cohen, West & Aiken (2003)

Factor Extension and the structure of affect

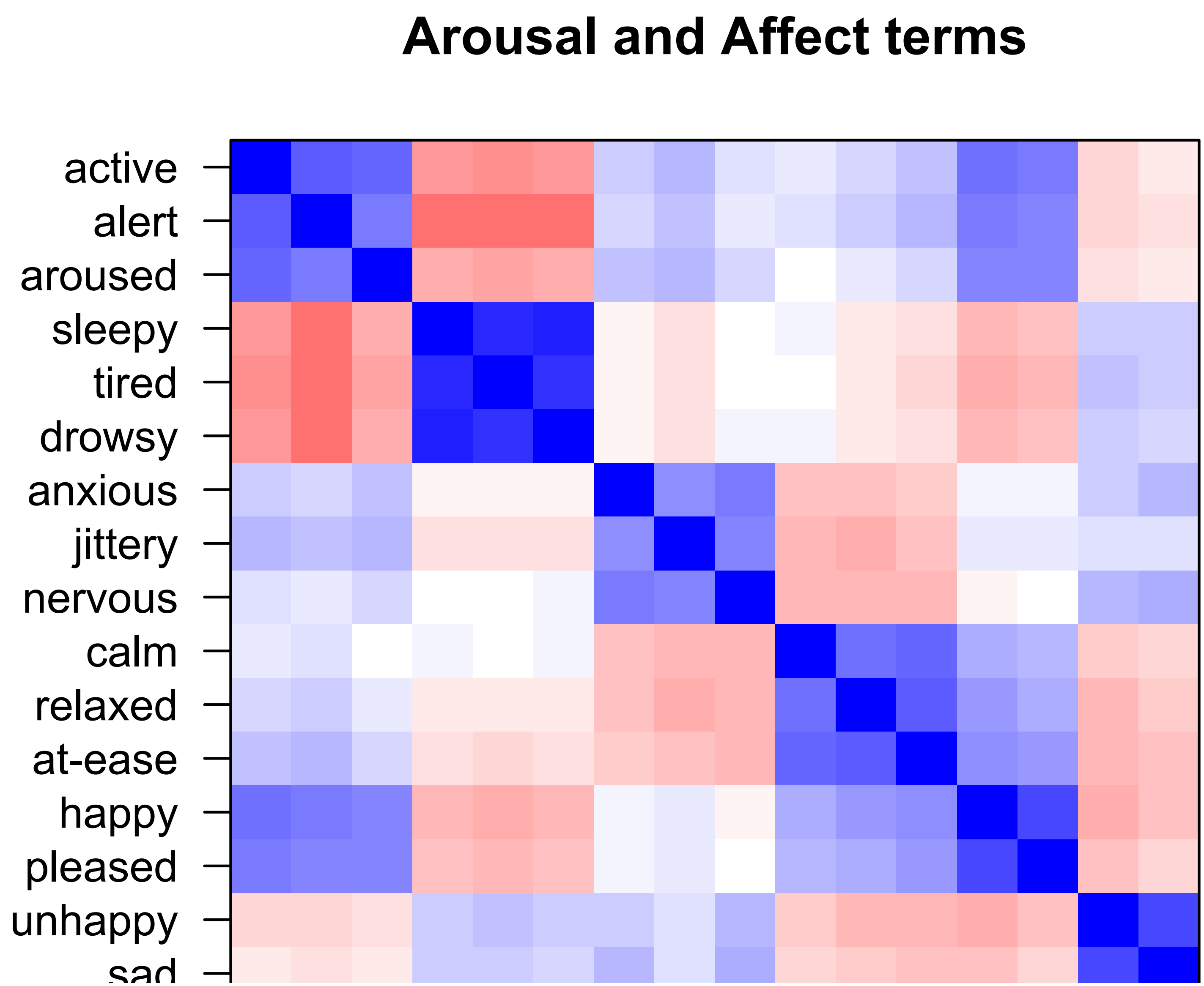
1. Consider the joint analysis of Energetic and Tense Arousal with Positive and Negative Affect
   - EA = "active" "alert" "aroused" -("sleepy" "tired" "drowsy")
   - TA = "anxious" "jittery" "nervous" -("calm" "relaxed" "at-ease")
   - PA = "happy" "pleased"
   - NA = "unhappy" "sad"
2. What is the location of PA and NA in the EA/TA space
3. What is the structure of the joint space?
4. Use the data in the Motivational State Questionnaire (msq) data set.
   - 75 mood and arousal items given over 10 years to various participants (N=3896)

Basic commands for display and and analysis

```
eata <- c("active","alert","aroused",
"sleepy","tired","drowsy",
"anxious","jittery","nervous",
"calm","relaxed","at-ease",
"happy","pleased","unhappy","sad")

R <- lowerCor(msq[eata])

cor.plot(R,main="Arousal and Affect terms")

f.all <- fa(R,2)
fe.all <- fa.extend(R,2,1:12,13:16)

op <- par(mfrow=c(1,2))
fa.plot(f.all,labels=rownames(R),ylim=c(-1,1),
    xlim=c(-1,1),title="FA combined")
fa.plot(fe.all,labels=rownames(R),ylim=c(-1,1),
     xlim=c(-1,1),title="Extend EA/TA")
```

1. get the data
2. find the correlations
3. show the correlations graphically
4. factor entire set
5. factor EA/TA space – extend to PA/NA
6. Display the results

A cor.plot of the data



Factor Extension and Set Correlation as ways of relating multiple domains

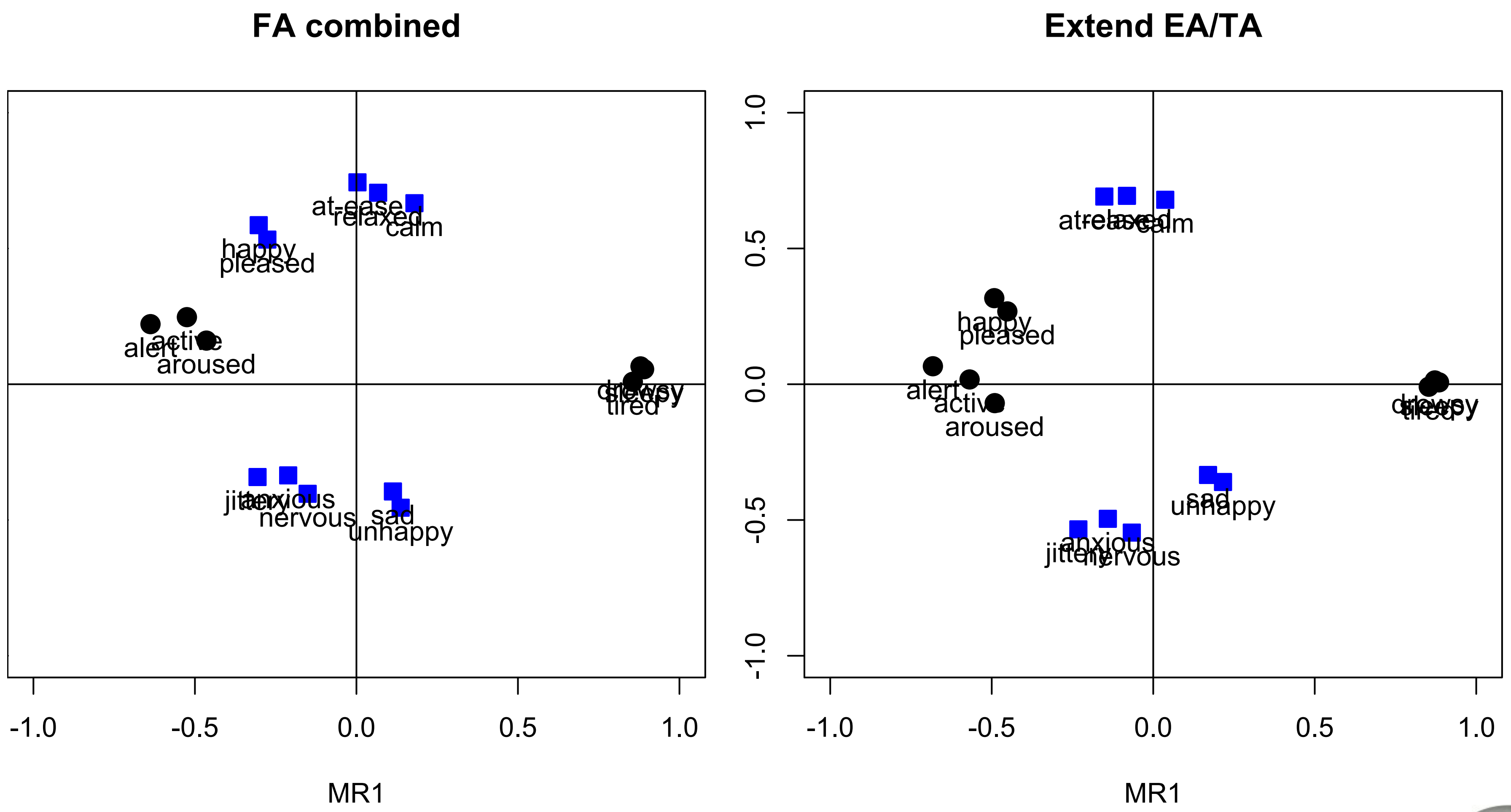
fa(r = R, nfactors = 2)

```
Factor Analysis using method =  minres
Call: fa(r = R, nfactors = 2)
Standardized loadings (pattern matrix)
          MR1   MR2   h2   u2
active  -0.52  0.25 0.39 0.61
alert   -0.64  0.22 0.52 0.48
aroused -0.46  0.16 0.27 0.73
sleepy   0.89  0.06 0.78 0.22
tired    0.86  0.01 0.73 0.27
drowsy   0.88  0.07 0.75 0.25
anxious -0.21 -0.34 0.13 0.87
jittery -0.31 -0.34 0.17 0.83
nervous -0.15 -0.40 0.16 0.84
calm     0.18  0.67 0.43 0.57
relaxed  0.07  0.71 0.48 0.52
at-ease  0.00  0.74 0.55 0.45
happy   -0.30  0.59 0.51 0.49
pleased -0.28  0.53 0.42 0.58
unhappy  0.14 -0.45 0.25 0.75
sad      0.11 -0.39 0.19 0.81

                       MR1  MR2
SS loadings           3.65 3.07
Proportion Var        0.23 0.19
Cumulative Var        0.23 0.42
Proportion Explained  0.54 0.46
Cumulative Proportion 0.54 1.00

 With factor correlations of
      MR1   MR2
MR1  1.00 -0.21
MR2 -0.21  1.00
```

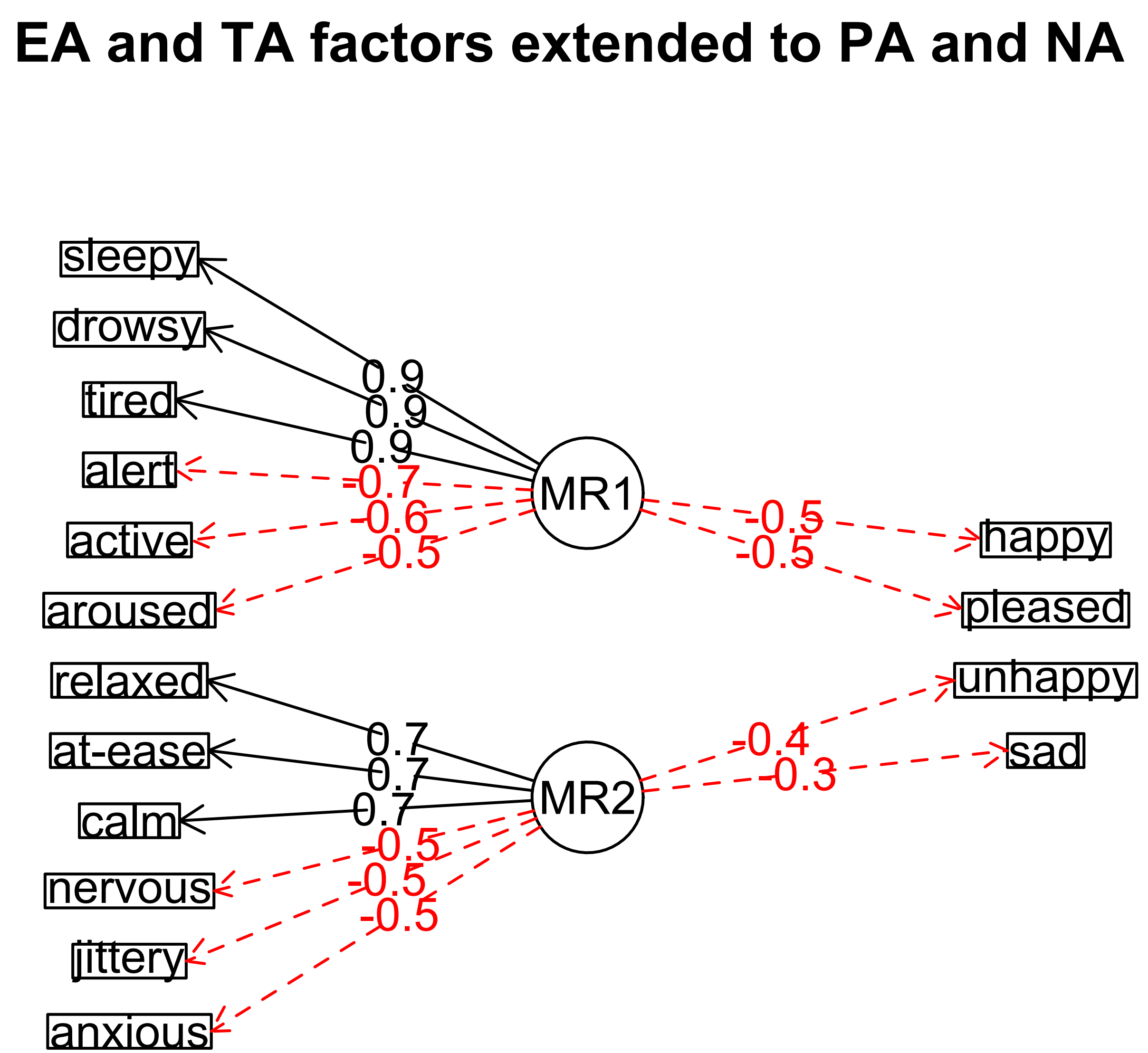
fa.extend(r = R, nfactors = 2, ov = 1:12, ev = 13:16)

```
Factor Analysis using method =  minres
Call: fa.extend(r = R, nfactors = 2, ov = 1:12, ev = 1
Standardized loadings (pattern matrix)
          MR1   MR2   h2   u2
active  -0.57  0.02 0.32 0.68
alert   -0.68  0.07 0.47 0.53
aroused -0.49 -0.07 0.24 0.76
sleepy   0.88  0.01 0.78 0.22
tired    0.85 -0.01 0.73 0.27
drowsy   0.87  0.01 0.76 0.24
anxious -0.14 -0.50 0.26 0.74
jittery -0.23 -0.53 0.33 0.67
nervous -0.07 -0.55 0.30 0.70
calm     0.04  0.68 0.46 0.54
relaxed -0.08  0.69 0.49 0.51
at-ease -0.15  0.69 0.51 0.49
happy   -0.49  0.32 0.36 0.64
pleased -0.45  0.27 0.29 0.71
unhappy  0.22 -0.36 0.19 0.81
sad      0.17 -0.33 0.15 0.85

                       MR1  MR2
SS loadings           3.95 2.69
Proportion Var        0.25 0.17
Cumulative Var        0.25 0.42
Proportion Explained  0.59 0.41
Cumulative Proportion 0.59 1.00

 With factor correlations of
      MR1   MR2
MR1  1.00 -0.06
MR2 -0.06  1.00
```

A fa.plot of the two solutions



Factor extension of Energetic and Tense Arousal to Affect



Set correlation is a generalized R^2 between two sets of variables

$R^2 = 1 - \prod(1 - \lambda_i^2)$ where λ_i^2 is the is ith squared canonical correlation. Unfortunately, the R^2 is sensitive to one of the canonical correlations being very high. An alternative, T^2, is the proportion of additive variance and is the average of the squared canonicals (Cohen et al., 2003).

```
> set.cor(y=13:16,x=1:12,data=R)

Call: set.cor(y = 13:16, x = 1:12, data = R)

Multiple Regression from matrix input

Beta weights
        happy pleased unhappy   sad
active   0.28    0.25   -0.07 -0.02
alert    0.17    0.15    0.05  0.01
aroused  0.16    0.20   -0.05 -0.04
sleepy   0.04    0.05    0.03  0.08
tired   -0.03   -0.05    0.17  0.14
drowsy   0.01    0.03    0.00 -0.04
anxious  0.01    0.01    0.10  0.17
jittery  0.02    0.00   -0.04 -0.03
nervous -0.01    0.01    0.19  0.20
calm     0.08    0.08    0.00  0.04
relaxed  0.13    0.10   -0.10 -0.06
at-ease  0.20    0.17   -0.12 -0.10
```

```
> set.cor(y=13:16,x=1:12,data=R)

Multiple R
  happy pleased unhappy     sad
   0.69    0.64    0.43    0.41

Multiple R2
  happy pleased unhappy     sad
   0.47    0.41    0.18    0.17

Various estimates of between set correlations
Squared Canonical Correlations
[1] 0.5187 0.1551 0.0095 0.0041
Chisq of canonical correlations
NULL

 Average squared canonical correlation =  0.17
 Cohen's Set Correlation R2 =  0.6
```

Classical Reliability Estimates

1. Guttman (1945) considered 6 different estimates of reliability. Of these, one λ_3 is also known as α (Cronbach, 1951).
2. McDonald (1999) introduced two additional reliability coefficients β which we (Zinbarg, Revelle, Yovel & Li, 2005; Revelle & Zinbarg, 2009) refer to as $\omega_{hierarchical}$ and ω_{total}.
   - $\omega_{hierarchical}$ or ω_h is an estimate of the general factor saturation of a test.
   - ω_{total} or ω_t is an estimate of the total reliable variance in a test.
3. All of these estimates of reliability are available in the *psych*.

α	`alpha`, `guttman`, `omega`, `score.items`
λ_{1-6}	`guttman`
ω_h, ω_t	`omega`

α can be misleading if applied to multifactorial items

Score the two dimensions of the Energetic and Tense Arousal items as one scale

```
> alpha(msq[eata[1:12]])

Reliability analysis
Call: alpha(x = msq[eata[1:12]])

  raw_alpha std.alpha G6(smc) average_r mean   sd
       0.76      0.74    0.84      0.19  1.1 0.32

 Reliability if an item is dropped:
        raw_alpha std.alpha G6(smc) average_r
active       0.71      0.69    0.82      0.17
alert        0.70      0.68    0.81      0.16
aroused      0.73      0.70    0.82      0.18
sleepy-      0.71      0.70    0.81      0.17
tired-       0.71      0.70    0.81      0.17
drowsy-      0.72      0.70    0.81      0.18
anxious      0.77      0.75    0.85      0.21
jittery      0.76      0.74    0.84      0.21
nervous      0.77      0.76    0.85      0.22
calm         0.78      0.76    0.85      0.22
relaxed      0.77      0.75    0.84      0.21
at-ease      0.76      0.74    0.84      0.20
```

```
 Item statistics
           n    r r.cor r.drop mean   sd
active  3890 0.73  0.70  0.627 1.03 0.93
alert   3885 0.78  0.77  0.714 1.15 0.91
aroused 3890 0.66  0.62  0.543 0.71 0.85
sleepy- 3880 0.69  0.71  0.620 1.25 1.05
tired-  3886 0.70  0.70  0.629 1.39 1.04
drowsy- 3884 0.67  0.68  0.600 1.16 1.03
anxious 2047 0.33  0.24  0.134 0.67 0.86
jittery 3890 0.37  0.29  0.189 0.59 0.80
nervous 3879 0.25  0.16  0.066 0.35 0.65
calm    3814 0.23  0.15  0.084 1.55 0.92
relaxed 3889 0.32  0.25  0.190 1.68 0.88
at-ease 3879 0.41  0.36  0.283 1.59 0.92

Non missing response frequency for each item
           0    1    2    3 miss
active  0.35 0.35 0.23 0.07 0.00
...
anxious 0.55 0.28 0.13 0.04 0.47
...
calm    0.13 0.35 0.35 0.17 0.02
relaxed 0.10 0.31 0.41 0.18 0.00
at-ease 0.13 0.33 0.37 0.17 0.00
Warning message:
In alpha(msq[eata[1:12]]) :
  Some items were negatively correlated with
  total scale and were automatically reversed
```

Compare α to ω_h for this multifactorial set of items

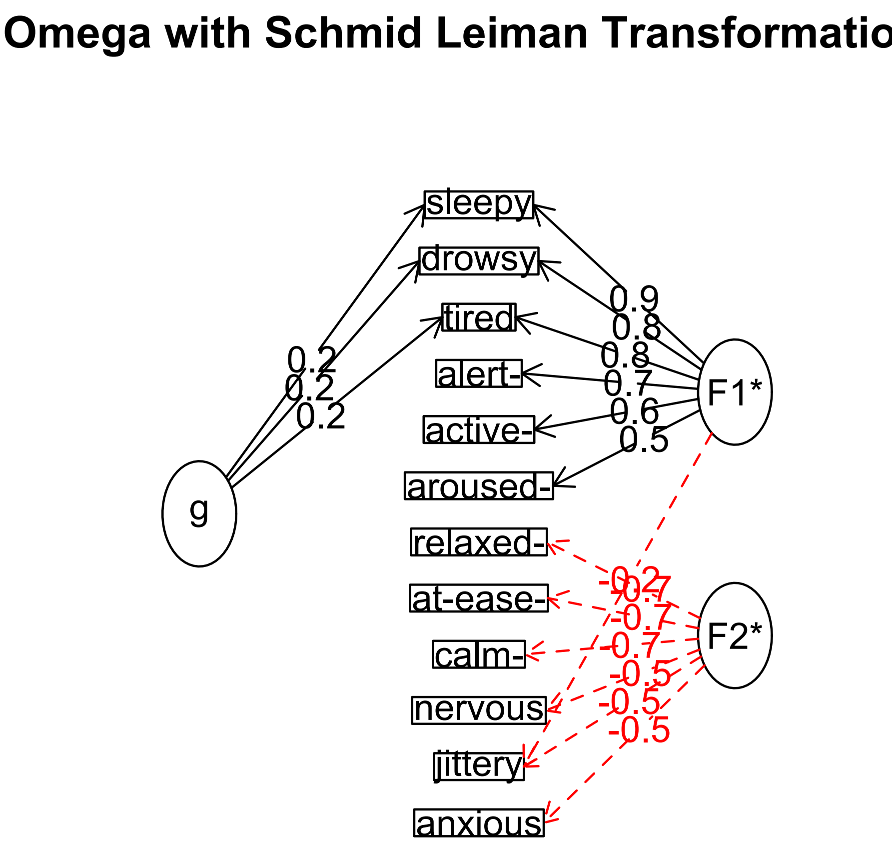
```
> omega(msq[eata[1:12]],2)

Omega
Call: omega(m = msq[eata[1:12]], nfactors = 2)
Alpha:                 0.75
G.6:                   0.85
Omega Hierarchical:    0.09
Omega H asymptotic:    0.11
Omega Total            0.83

Schmid Leiman Factor loadings greater than  0.2
             g   F1*   F2*   h2   u2   p2
active-          0.55        0.32 0.68 0.06
alert-           0.66        0.47 0.53 0.07
aroused-         0.48        0.24 0.76 0.04
sleepy    0.21   0.86        0.78 0.22 0.06
tired     0.20   0.83        0.73 0.27 0.06
drowsy    0.20   0.85        0.76 0.24 0.05
anxious               -0.48 0.26 0.74 0.03
jittery         -0.23 -0.52 0.33 0.67 0.02
nervous               -0.53 0.30 0.70 0.04
calm-                 -0.66 0.46 0.54 0.05
relaxed-              -0.67 0.49 0.51 0.07
at-ease-  0.20        -0.67 0.51 0.49 0.08

With eigenvalues of:
   g  F1*  F2*
0.31 3.22 2.13
```

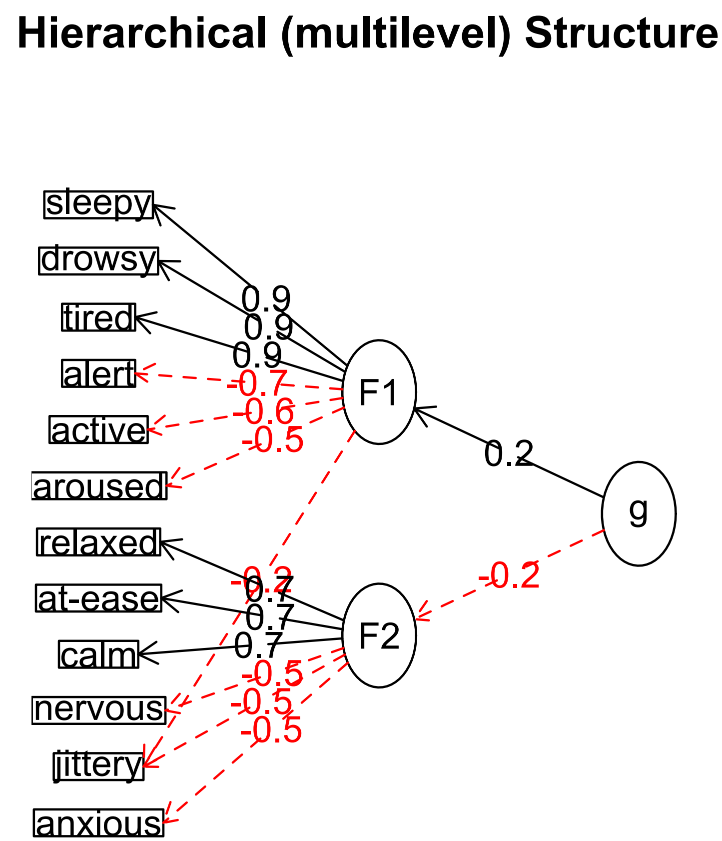
1. ω_h is a higher order factor model and requires 3 lower level factors for identification.
2. It can be found with two factors under various assumptions.
3. By default, omega assumes equal loadings of the lower level factors on the higher order factor, but this may be changed.
4. A warning is given for this condition.

Representing a higher order structure

1. *omega$_h$* may be found by Exploratory Factor Analysis by factoring the data, applying an oblique transformation (e.g., `oblimin`) and then factoring the correlation matrix of these resulting factors. Factor loadings on the *general factor* are then found using the Schmid & Leiman (1957) transformation.
2. Alternatively, *omega$_h$* may be directly estimated using Confirmatory Factor Analysis using the *sem* (Fox, Nie & Byrnes, 2012) or *lavaan* (Rosseel, 2012) packages.
3. `omegaSem` performs an EFA and then passes the resulting solution to *sem* to do the CFA. Unfortunately, for the two factor condition, the solution is not defined.
4. The graphical representation of the Schmid-Leiman transformation is automatically drawn by `omega`.

This shows that there is no general factor of these two dimensions





Consider another example: 16 ability items

1. 16 ability items reflecting 4 subdomains for N=1525.
2. Example is taken from `iqitems` in *psych*.
3. Collected using SAPA (Synthetic Aperture Personality Assessment) as part of the ICAR (International Cognitive Ability Resource) project.
4. Convert multiple choice to Correct/Incorrect
5. Score for traditional α using `alpha` as well as ω_h.

Finding α and ω_h for 16 ability items

```
> data(iqitems)

> iq.keys <- c(4,4,4, 6,  6,3,4,4,
        5,2,2,4,   3,2,6,7)

> score.multiple.choice(iq.keys,iqitems)

> iq.scrub <- scrub(iqitems,isvalue=0)

> iq.tf <-  score.multiple.choice(
   iq.keys,iq.scrub,  score=FALSE)

> alpha(iq.tf)

> omega(iq.tf,nfactors=4)
```

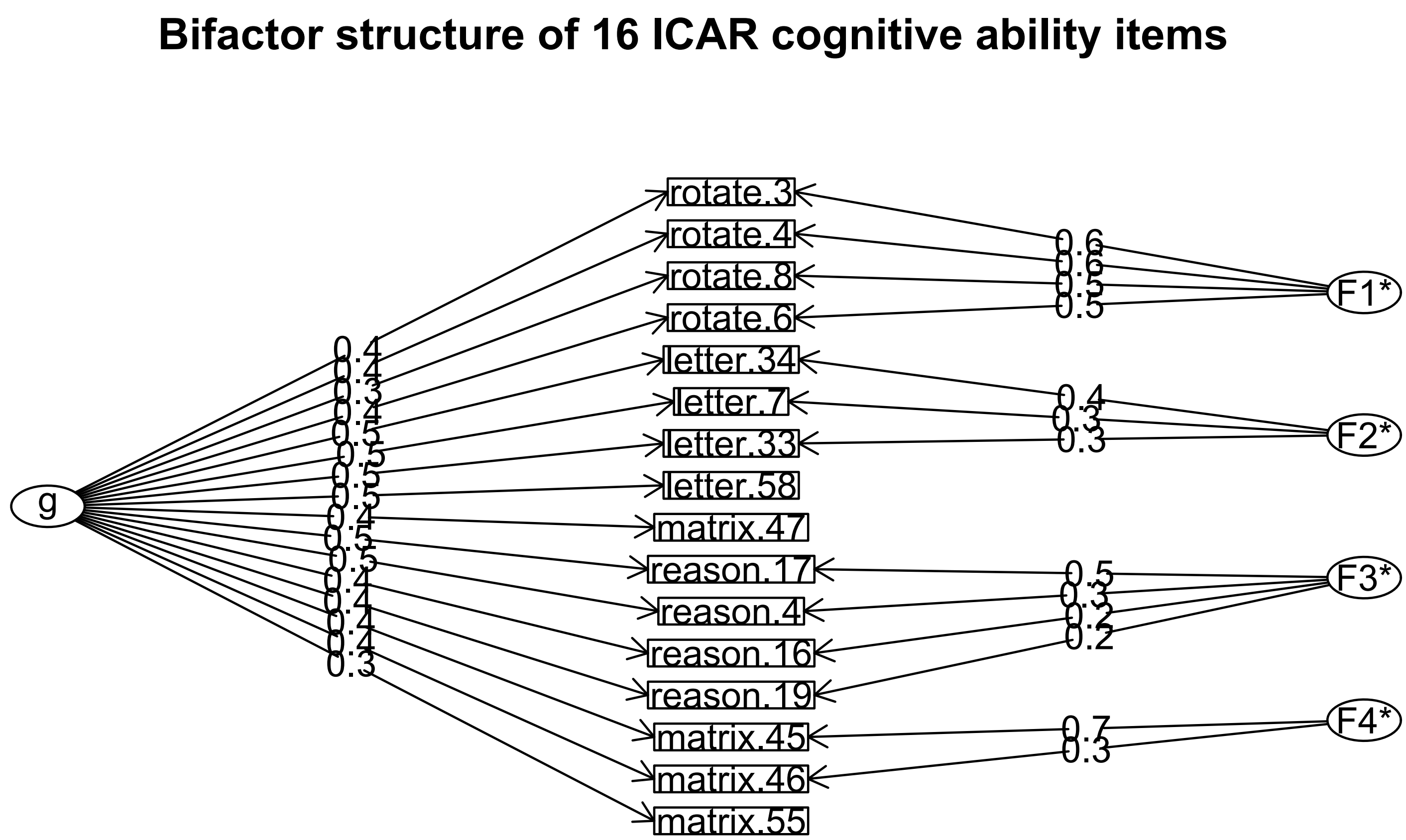
1. Get the data
2. Assign a scoring key
3. Score the items to get summary statistics
4. Convert non-responses to missing (NA)
5. Convert the multiple choice items to correct/incorrect
6. Find conventional α
7. Find ω_h

Comparing α and ω_h for hierarchically organized data

```
> alpha(iq.tf)

Reliability analysis
Call: alpha(x = iq.tf)

  raw_alpha std.alpha G6(smc) average_r mean   sd
      0.83      0.83    0.84      0.23 0.49 0.25

 Reliability if an item is dropped:
          raw_alpha std.alpha G6(smc) average_r
reason.4       0.82      0.82    0.82      0.23
...
rotate.8       0.82      0.82    0.83      0.24

 Item statistics
             n    r r.cor r.drop mean   sd
reason.4  1442 0.58  0.54   0.50 0.68 0.47
...
rotate.8  1460 0.51  0.47   0.41 0.19 0.39

Non missing response frequency for each item
             0    1 miss
reason.4  0.32 0.68 0.05
...
rotate.8  0.81 0.19 0.04
```

```
> omega(iq.tf,nfactors=4)

Omega
Call: omega(m = iq.tf, nfactors = 4)
Alpha:                 0.83
G.6:                   0.84
Omega Hierarchical:    0.65
Omega H asymptotic:    0.76
Omega Total            0.86

Schmid Leiman Factor loadings greater than  0.2
                g   F1*   F2*   F3*   F4*   h2   u2   p2
reason.4  0.50              0.27        0.34 0.66 0.73
reason.16 0.42              0.21        0.23 0.77 0.76
reason.17 0.55              0.47        0.52 0.48 0.57
reason.19 0.44              0.21        0.25 0.75 0.77
letter.7  0.52        0.35              0.39 0.61 0.69
letter.33 0.46        0.30              0.31 0.69 0.70
letter.34 0.54        0.38              0.43 0.57 0.67
letter.58 0.47        0.20              0.28 0.72 0.78
matrix.45 0.40                    0.66  0.59 0.41 0.27
matrix.46 0.40                    0.26  0.24 0.76 0.65
matrix.47 0.42                          0.23 0.77 0.79
matrix.55 0.28                          0.12 0.88 0.65
rotate.3  0.36  0.61                    0.50 0.50 0.26
rotate.4  0.41  0.61                    0.54 0.46 0.31
rotate.6  0.40  0.49                    0.41 0.59 0.39
rotate.8  0.32  0.53                    0.40 0.60 0.26

With eigenvalues of:
   g  F1*  F2*  F3*  F4*
```

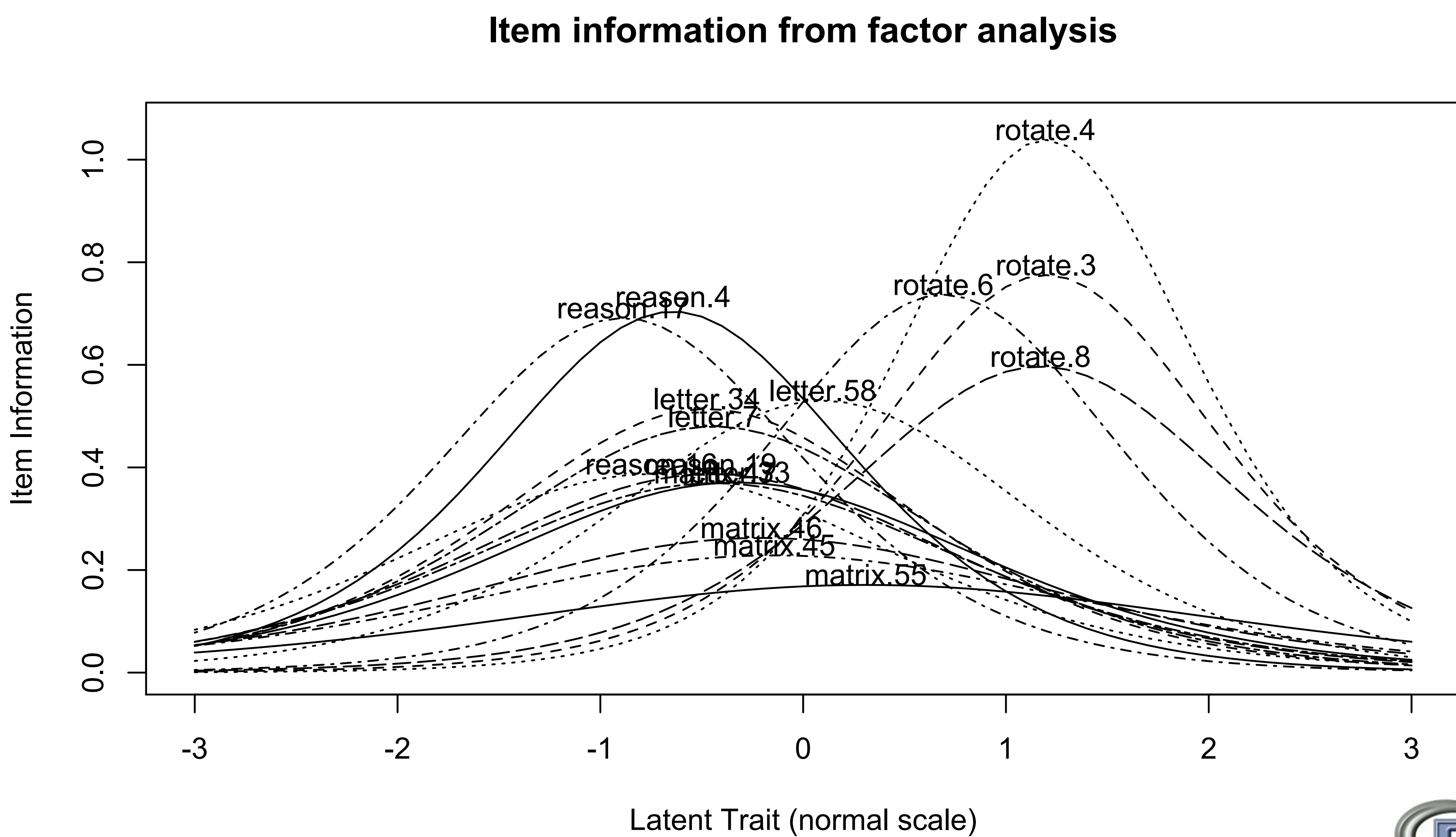
Bifactor solution to the 16 ICAR ability items shows g and first order factors



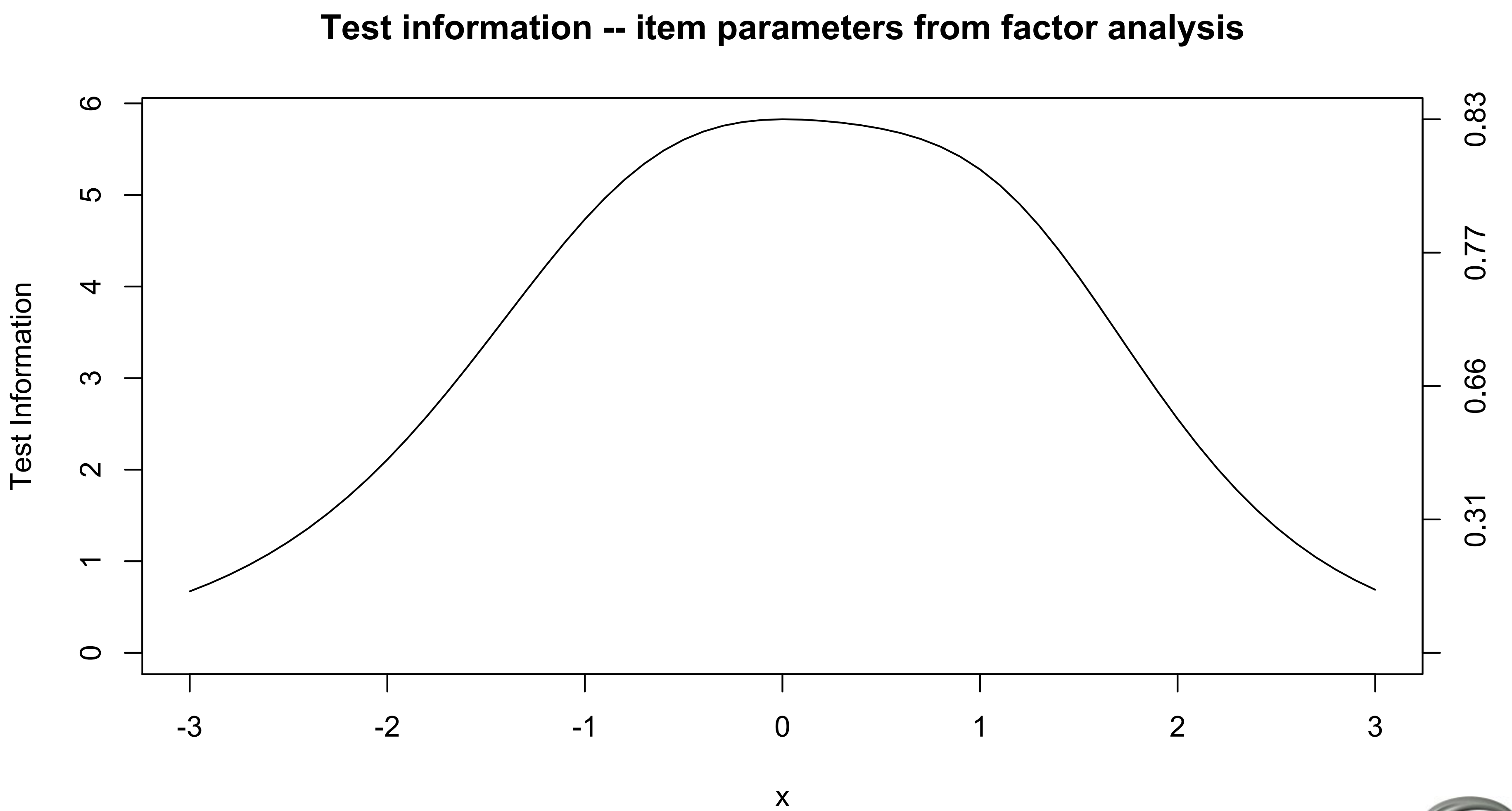
2 parameter IRT is equivalent to EFA solution

1. Item Response Theory approaches consider item difficulty and item discrimination.
    - 1 parameter IRT considers just item location and applies the Rasch model. Can be found using the *ltm* package.
    - 2 parameters of IRT are location and discrimination. These are reparameterizations of factor loadings and item difficulty: That is, 2 parameter IRT models are just factor models applied to the *tetrachoric* or *polychoric* correlations.
        - That is, find the factor analysis loadings (λ_i) and the item endorsement frequencies expressed as normal deviates (τ_i and then convert to IRT parameters
        - discrimination $\alpha = \frac{\lambda_i}{\sqrt{1-\lambda_i^2}}$
        - location (difficulty) $\delta = \frac{\tau_i}{\sqrt{1-\lambda_i^2}}$
2. IRT statistics can be done using `irt.fa`.

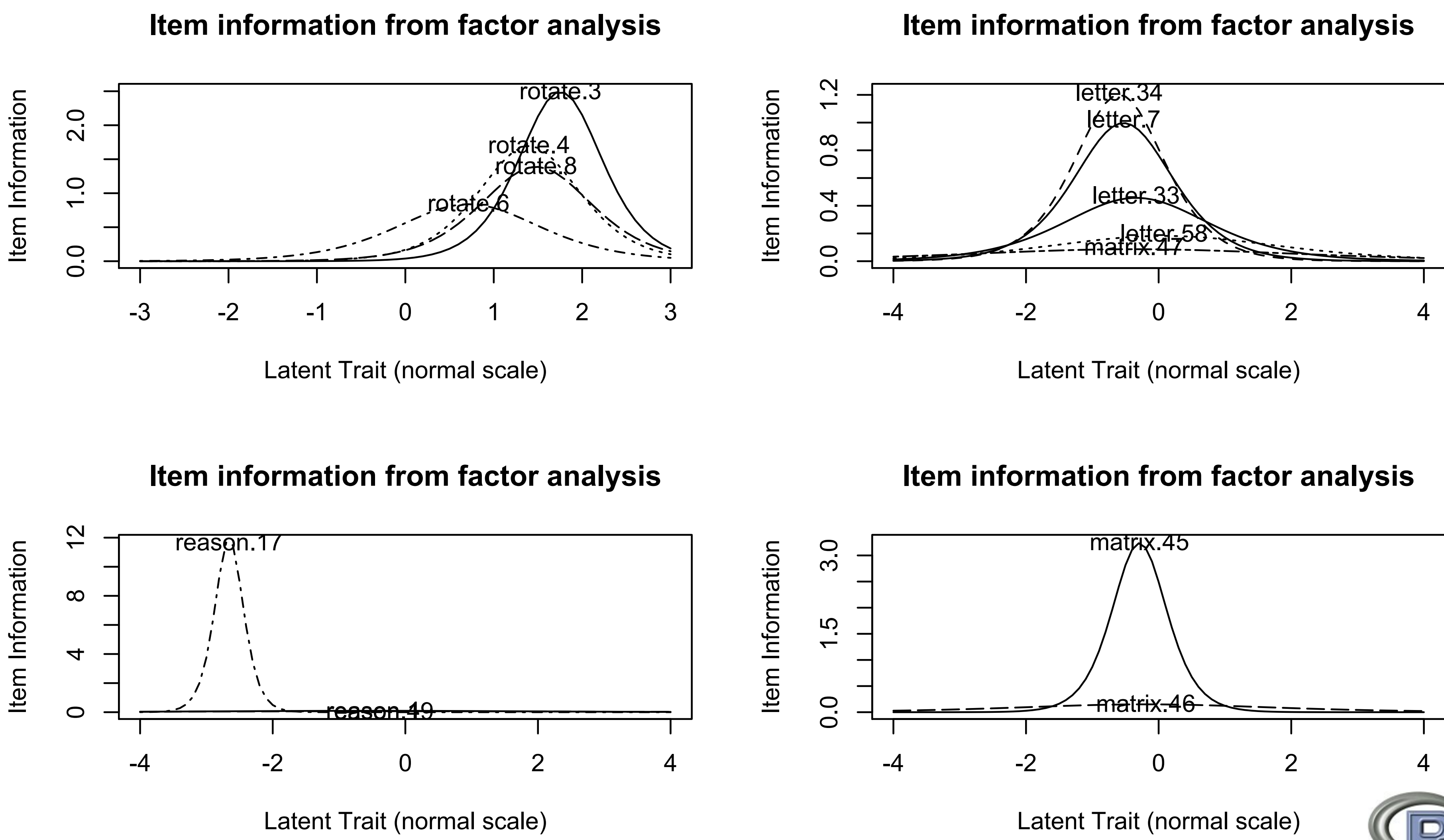
Item information for a 1 factor solution



Test information for a 1 factor solution



Item information for each lower level factor of 16 ICAR items



Using R for personality research: Classical and Modern psychometrics

Combining the power of base R with additional packages allows personality researchers to

1. Do basic scale construction
2. Perform classical (α) and more advanced (ω_h, ω_t) analyses of reliability.
3. Perform Exploratory and Confirmatory Factor Analysis
4. Do "modern" psychometrics using Item Response Theory